Experiments on Viscous Transport in Pure-Electron Plasmas

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Abstract. Viscous transport in pure-electron plasmas is a rearrangement of particles due to like-particle interactions, eventually leading to a confined global thermal equilibrium state. The measured transport is observed to be proportional to the shear in the total ($\mathbf{E} \times \mathbf{B}$ + diamagnetic) fluid rotation of the plasma, for both hollow and monotonic rotation profiles. We determine the local kinematic viscosity, κ, from measurements of the local flux of electrons. The measured viscosity is $50 - 10^4$ times larger than expected from classical transport due to short-range velocity-scattering collisions, but is within a factor of 10 of recent theories by O'Neil and Dubin of transport due to long-range drift collisions. The measured viscosity scales with magnetic field and plasma length roughly as $\kappa \propto B/L$. This scaling suggests a finite-length transport enhancement caused by particles interacting multiple times as they bounce axially between the ends of the plasma.

INTRODUCTION

Like-particle interactions cause a non-neutral plasma to evolve toward the global thermal equilibrium state of rigid rotation and essentially uniform density. This cross-field transport to thermal equilibrium was previously observed in 1988 by Driscoll *et.al.* using short hollow plasmas [1]. The global rate at which the plasma approached equilibrium was found to be orders of magnitude faster than classical theory predictions and to scale differently with magnetic field.

In this paper, we summarize recent detailed measurements of like-particle transport in electron plasma columns [2]. The radial flux of electrons is well described by a local model of viscosity, in which the transport is driven by local shears in the fluid velocity of the column. From measured density profiles we experimentally determine the local coefficient of viscosity and compare the results to theoretical predictions. We find that recent *Long-Range* theories, which consider interaction distances on the order of a Debye length, are in much better agreement with the measurements than *Classical* theory, which only considers interaction distances on the order of a cyclotron radius.

THEORIES OF VISCOUS TRANSPORT

Comparisons between measurements and theory are based on a standard model of viscous transport in a cylindrically-symmetric fluid [3]. Here, we are concerned with the radial transport due to shears in the azimuthal velocity. This transport is described by the (r, θ) component of the stress tensor,

$$P_{r\theta} = -\eta\, r \frac{\partial \omega_{tot}}{\partial r}, \tag{1}$$

where η is the coefficient of viscosity and $\omega_{tot} \equiv \omega_E + \omega_D = \frac{c}{B\,r}\left[\frac{\partial \phi}{\partial r} - \frac{1}{e\,n}\frac{\partial (n\,T)}{\partial r}\right]$ is the total ($\mathbf{E} \times \mathbf{B} +$ diamagnetic) rotation frequency in the azimuthal direction.

The viscosity coefficient, in effect, describes angular momentum exchange between interacting particles. It has the form $\eta = m_e\, n\, \nu_{eff}\, \delta^2$, where ν_{eff} is the effective collision frequency of momentum exchange, and δ is the distance over which the electrons interact. In this paper, we plot the kinematic viscosity κ, which is η scaled by the electron density n and mass m_e,

$$\kappa \equiv \frac{\eta}{m_e\, n} = \nu_{eff}\, \delta^2. \tag{2}$$

A simplified summary of theoretical predictions for κ is displayed in the table below.

Classical theory [4] describes transport due to velocity-scattering collisions between electrons separated by a distance on the order of the cyclotron radius (*i.e.* $\delta \sim r_c$), as shown in Figure 1(a). These interactions occur at an effective rate on the order of the electron-electron collision frequency, *i.e.* $\nu_{eff} \sim \nu_{ee} \approx \nu_c \ln(r_c/b)$, where $\nu_c \equiv n\, \bar{v}\, b^2$, $\bar{v}$ is the thermal velocity, and b is the distance of closest approach.

Long-Range theories by Dubin and O'Neil [5–7] describe transport due to $\mathbf{E} \times \mathbf{B}$ drift collisions, as shown in Figure 1(b). The "3-D" version of the theory [5] considers an infinite length plasma, so electrons effectively collide only once. The interaction distance is a Debye length, *i.e.* $\delta \sim \lambda_D$, and the effective collision frequency is $\nu_{eff} \sim \nu_c \ln(\omega_p/\nu_{ee})$, where ω_p is the plasma frequency. In a non-neutral plasma, this *Long-Range* prediction gives a substantially larger viscosity coefficient than *Classical* theory, since $\lambda_D \gg r_c$.

		Interaction Distance δ	Effective Collision Frequency ν_{eff}	Scaling of κ with B-field & Length
Classical		r_c	$\nu_c \ln(r_c/b)$	$B^{-2}\, L^0$
Long-Range	$3D$	λ_D	$\nu_c \ln(\omega_p/\nu_{ee})$	$B^0\, L^0$
	$2D$	$\lambda_D\, r_c\, \omega_p \left\vert \frac{1/L\ \partial L/\partial r}{r\ \partial \omega_E/\partial r} \right\vert$	$\nu_c\, N_{coll}$	$B^1\, L^{-3}$
Empirical		λ_D	$\nu_c\,(1 + N_{coll})$	$B^1\, L^{-1}$

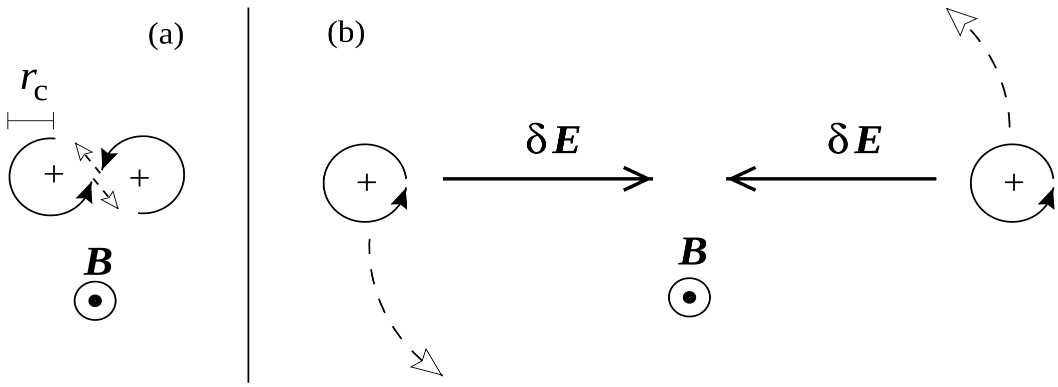

FIGURE 1. Cartoon of electron-electron interactions as described by (a) *Classical* velocity-scattering theory (b) *Long-Range* $\mathbf{E} \times \mathbf{B}$ drift theory.

Enhancements to the *Long-Range* viscosity are predicted to occur when finite length effects are included in the theory; but these effects are not yet understood in detail. As electrons bounce axially between the ends of the plasma (with thermal rate $\overline{f_b} \equiv \bar{v}/2L$), they "collide" $N_{coll} \equiv \overline{f_b}/|r\,\partial\omega_E/\partial r|$ times before being sheared apart. In the current "2-D" *Long-Range* theory [7], the bounce motion is averaged out and electrons are considered to interact as rods of charge. The interaction distance is listed in the table and the effective collision frequency is $\nu_{eff} \sim \nu_c\, N_{coll}$. This theory applies to plasmas with either monotonic or hollow rotation profiles, whereas a previous version [6] only applied to hollow plasmas.

In the table, we also list an empirical viscosity formula, which uses the interaction distance of the 3-D *Long-Range* theory ($\delta = \lambda_D$) and an effective collision frequency similar to that of the 2-D *Long-Range* theory ($\nu_{eff} = \nu_c\,(1 + N_{coll})$). We find that this simple empirical viscosity provides the best agreement with the measurements.

MEASURED VISCOSITY

We determine the radial particle transport from measurements of the density profile of the plasma $n(r)$. The local radial flux of particles, $\Gamma(r) \equiv -\frac{1}{r}\int_0^r dr'\, r'\, \frac{\Delta n(r')}{\Delta t}$, is calculated from the change in density $\Delta n(r)$ over a duration Δt, and the local experimental stress (or flux of angular momentum) is $P_x(r) \equiv \frac{eB}{c}\,\frac{1}{r^2}\int_0^r dr' r'^2\, \Gamma(r')$. Motivated by the viscous model, the kinematic viscosity is then calculated experimentally as the ratio of the measured stress to the shear in the plasma $\kappa_x(r) \equiv= \frac{P_x(r)}{-m_e\, n(r)\, r\, \partial\omega_{tot}(r)/\partial r}$, where $\omega_{tot}(r)$ is determined from the measured density $n(r)$ and temperature T.

Figure 2 shows the evolution of the radial density and rotation profile at three different times. Initially, the plasma has substantial density gradients and rotational shear. The plasma rearranges itself to eliminate the shears; as a consequence, some

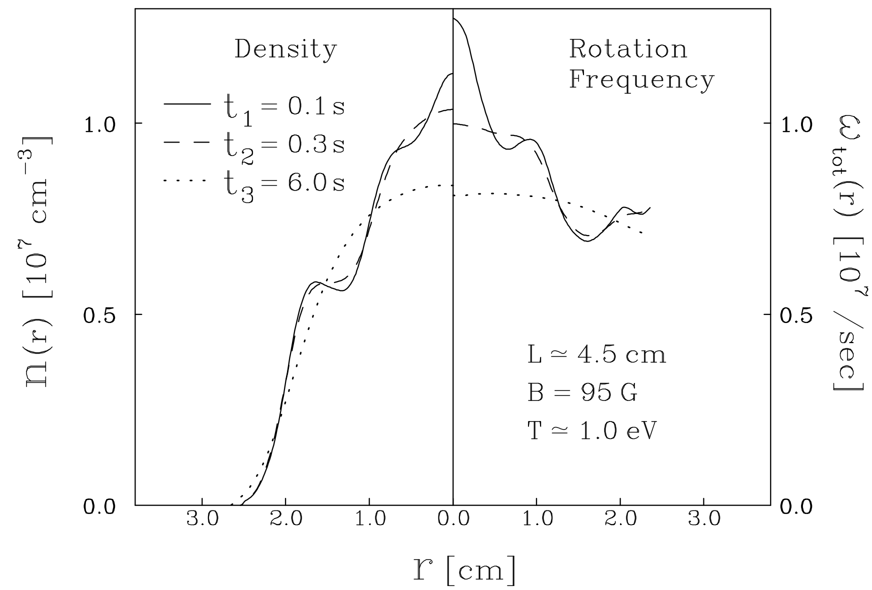

FIGURE 2. Example of density profile evolution due to viscous transport.

electrons move radially inward while others move outward. The angular momentum is conserved throughout the entire evolution shown in Figure 2, indicating the transport is due to like-particle (internal) interactions. In addition, the local flux of particles appears to be driven by the local shears in the *total* plasma rotation, in verification of the viscous model [2].

The local kinematic viscosity $\kappa_x(r)$ is calculated for small values of Δt so that the plasma parameters do not vary substantially, *e.g.* between $t_1 = 0.1$ s and $t_2 = 0.3$ s in Figure 2. We then average $\kappa_x(r)$ over the radial region near the density peak (where the signal to noise is best) to obtain a single number κ_x for any given set of initial conditions.

Measurements of the kinematic viscosity for relatively short plasmas are shown in Figure 3 as a function of the confining magnetic field. The measured viscosity is as much as 4 orders of magnitude larger than predictions from *Classical* theory and increases with magnetic field roughly as $\kappa_x \propto B^1$. Predictions of the 3D *Long-Range* theory, while much closer than *Classical* theory, are smaller than the measured values and scale as $\kappa \propto B^0$. The 2D *Long-Range* theory prediction has the B^1 scaling, but is about 10 times larger. An additional result is that we find no substantial difference between the measured viscosity for a hollow profile and that for a monotonic profile.

We have measured the viscosity for different length plasmas, and find the approximate scaling $\kappa_x \propto 1/L$ [2]. (Note: This scaling is in sharp contrast to the observed increase in (external) asymmetry-induced transport with plasma length for these "slightly-rigid" plasmas [2,8].) The viscosity coefficient also appears to depend upon the $\mathbf{E} \times \mathbf{B}$ shear in the plasma, as described by N_{coll}.

The entire parameter dependence is summarized by Figure 4, which displays the

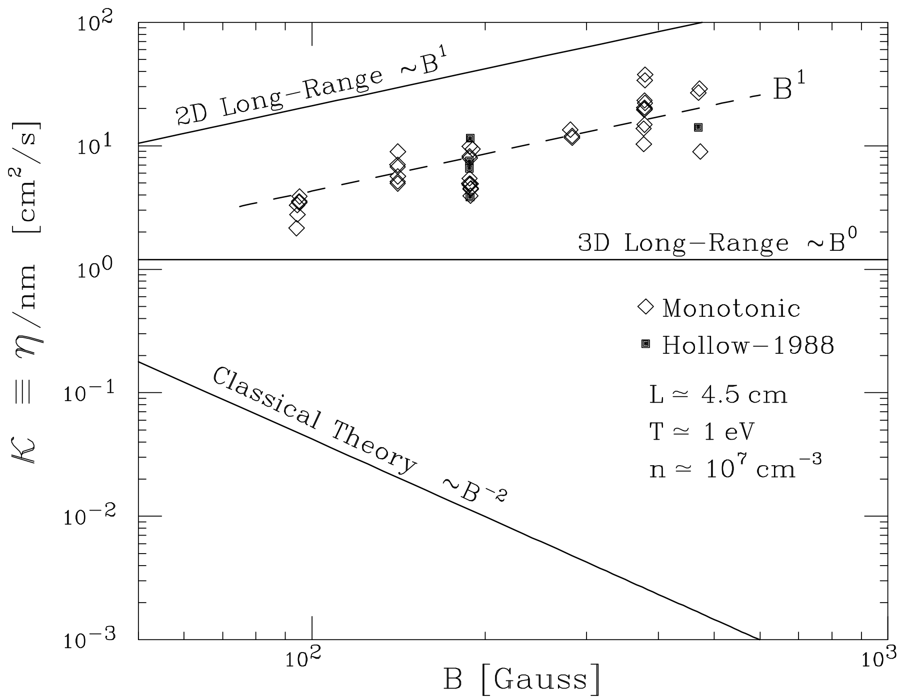

FIGURE 3. Kinematic viscosity, κ, versus magnetic field, B.

scaled kinematic viscosity $\kappa/\nu_c\lambda_D^2$ as a function of the average number of collisions N_{coll}. The data roughly follows the simple empirical formula

$$\kappa = (1 + N_{coll})\, \nu_c\, \lambda_D^2. \tag{3}$$

In terms of the rigidity $\mathcal{R} \equiv 2\pi\,\overline{f_b}/\omega_E$, the plasmas used in this study were "slightly-rigid" with $1 \lesssim \mathcal{R} \lesssim 10$. The prediction from the 3D *Long-Range* theory is in agreement with the measured data for $N_{coll} \approx 1$, where the plasma is somewhat "floppy" ($\mathcal{R} \lesssim 1$) and the enhancement due to multiple collisions is weak. At larger values of N_{coll}, the plasma is "slightly-rigid" and the measured viscosity is larger than the 3-D predictions by an amount that is approximately proportional to N_{coll}. Further experiments are planned using "highly-rigid" plasmas ($\mathcal{R} \gg 10$) at larger magnetic fields (up to $B = 10^4$ G) to obtain the viscosity at even larger values of N_{coll}.

In other experiments, the 3-D *Long-Range* theory of like-particle interactions has successfully predicted coefficients for heat [9] and test-particle transport [10]. Both these studies were conducted on "floppy" ion plasmas, for which $N_{coll} \leq 1$. Experiments have not yet tested the 2-D *Long-Range* predictions [11] for these coefficients at higher values of N_{coll}.

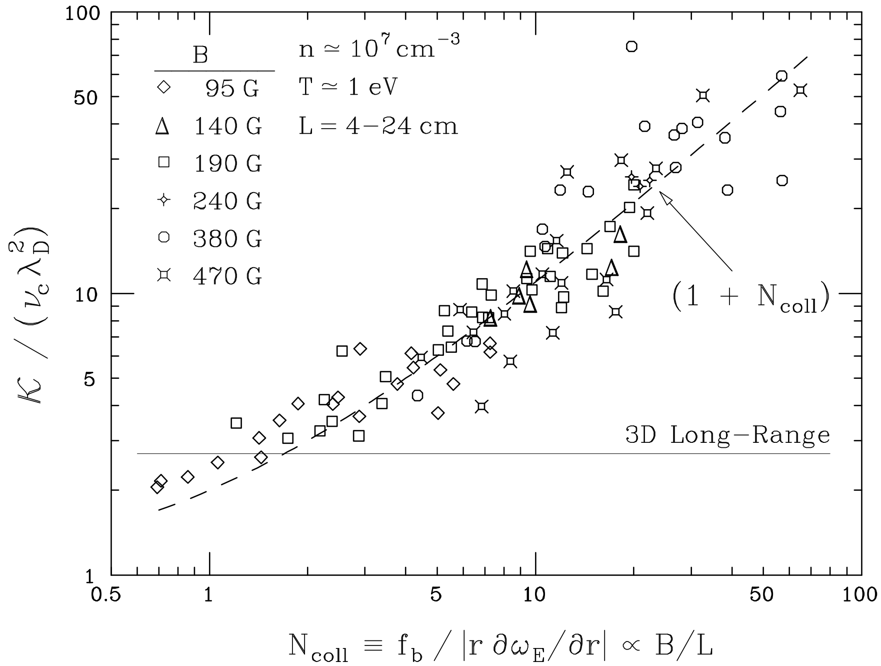

FIGURE 4. Scaled kinematic viscosity versus the average number of collisions, N_{coll}.

SUMMARY

We determine the effective viscosity in electron plasmas from measurements of the local flux of particles. The measured viscosity disagrees strongly with *Classical* velocity-scattering theory, but agrees well with 3-D *Long-Range* $\mathbf{E} \times \mathbf{B}$ drift theory when the plasma is somewhat "floppy". At higher plasma rigidity, finite length effects appear to cause an enhancement to the viscosity which scales roughly as the average number of collisions $N_{coll} \propto B/L$. A 2-D theory predicts such an enhancement, but gives viscosity coefficients about 10 times too large.

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