Cross-Magnetic-Field Heat Conduction in Non-neutral Plasmas

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This Letter discusses cross-magnetic-field collisional heat transport for a non-neutral plasma in the typical operating regime $\lambda_D \gg r_c$, where λ_D is the Debye length and r_c is the cyclotron radius. The dominant transport mechanism is the exchange of energy associated with velocity components parallel to the magnetic field. For a thermal gradient scale length $L_T \gtrsim 100\lambda_D$, the energy exchange is dominated by interactions between particles separated by $O(\lambda_D)$ and yields a thermal diffusivity $\chi \sim \nu_c \lambda_D^2$, where $\nu_c = n\bar{v}b^2$ is the collision frequency. The diffusivity is even larger for larger L_T, where the energy exchange is dominated by the emission and absorption of plasma waves. [S0031-9007(97)03015-9]

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Magnetically confined non-neutral plasma experiments typically operate in the parameter regime $r_c \ll \lambda_D$, where $r_c = \bar{v}/\Omega_c$ is the cyclotron radius and $\lambda_D = \sqrt{T/4\pi ne^2}$ is the Debye length. (Here, T is the temperature, $\bar{v} = \sqrt{T/m}$ is the thermal speed, n is the density, and $\Omega_c = eB/mc$ is the cyclotron frequency.) This Letter contains a calculation of the cross magnetic field thermal conductivity for such a plasma.

One might be tempted to apply the well-known "classical" expression for thermal conductivity [1], $\kappa^{\mathrm{class}} = (8/3)\sqrt{\pi}\, n\nu_c r_c^2 \ln(r_c/b)$, where $\nu_c = n\bar{v}b^2$ is the frequency of large angle scatterings and $b = e^2/T$ is the distance of closest approach. However, this expression is not relevant in the regime $r_c \ll \lambda_D$. The classical theory envisions a transport mechanism where binary collisions produce the scattering of particle velocity vectors which then results in cross-field steps $\Delta x \sim r_c$ of the particle guiding centers.

The classical thermal diffusivity, $\chi^{\mathrm{class}} = \kappa^{\mathrm{class}}/(5/2n)$, is then estimated as $\chi^{\mathrm{class}} \sim \nu(\Delta x)^2 \sim \nu r_c^2$, where here the interaction rate $\nu = \nu_c \ln(r_c/b)$ is the collision frequency. The logarithm enters through an integral over impact parameter that is cut off at the lower limit $\rho = b$ and the upper limit $\rho = r_c$. The upper cutoff reminds us that velocity scatterings occur only for collisions that have impact parameter $\rho < r_c$.

However, for $r_c \ll \lambda_D$, many collisions have an impact parameter in the range $\lambda_D > \rho \gg r_c$, and such collisions do not scatter the velocity vectors. Indeed, to lowest order in r_c/ρ, the adiabatic invariant $\mu = mv_\perp^2/2B$ is conserved for each particle, so the collision reduces to a one-dimensional elastic collision. Nevertheless, energy exchange can occur through the interchange of parallel velocities (as in any 1D collision), and this process dominates the heat transport. For interactions that are Debye shielded, the energy exchange can occur over a distance $\Delta x \sim \lambda_D$, and the thermal diffusivity is of order $\chi^{\mathrm{Debye}} \sim \nu_c \lambda_D^2$. This is independent of magnetic field strength and is much larger than the classical diffusivity in the parameter regime $r_c \ll \lambda_D$.

We need not look at the transport solely through the perspective of binary collisions. Rather, we can understand that charges on one field line make stochastic fields (thermal fluctuations) that accelerate charges on a spatially separated field line. If the separation is of order $\rho \sim \lambda_D \gg r_c$, then the frequency of the stochastic fields is of order $\omega \sim \omega_p \ll \Omega_c$, so the cyclotron action is conserved and only parallel acceleration is significant. From this perspective, it is clear that the interaction distance Δx need not be limited to λ_D.

The Cherenkov emission of a plasma wave by a particle and its absorption by another particle at a different location can allow transfer of energy over larger distances. Now $\Delta x \sim L_T$, the thermal gradient scale length, and we expect $\chi^{\mathrm{waves}} = \nu L_T^2$. However, the interaction rate ν is no longer the collision frequency. Rather, we can estimate ν as the rate that lightly damped waves are excited, multiplied by the fractional energy density in these waves compared to the plasma thermal energy $3NT/2$. Detailed balance implies the excitation rate equals the damping rate γ of a wave that damps over a distance L_T. The damping rate is related to L_T by $\gamma = V_g/L_T$, where V_g is the wave group velocity, which one can take to be of order $\bar{v}$ for plasma waves. The energy in each wave is $T/2$, and the number of waves is approximately the density of states $\Lambda = V/(2\pi)^3$, where V is the volume of the plasma, multiplied by the k-space volume of waves with $|\mathbf{k}| \lesssim \alpha\lambda_D^{-1}$, where α is a number of order $1/3$ or less (since only such waves are lightly damped).

Thus, we obtain the following estimate for the rate of energy transfer by waves $\nu \sim \gamma(T/2) \times [4\pi(\alpha\lambda_D^{-1})^3/3]\Lambda/(3NT/2)$. The thermal diffusivity due to waves is then

$$\chi^{\mathrm{waves}} \sim \alpha^3 \nu_c \lambda_D L_T \,. \quad (1)$$

Note that $\chi^{\mathrm{waves}} > \chi^{\mathrm{Debye}}$ when $L_T \gtrsim \lambda_D/\alpha^3$. This rough estimate will be refined below. It shows, however, that lightly damped plasma waves play an important role in the thermal conductivity provided that L_T is sufficiently large. This wave mechanism was discussed originally by Rosenbluth and Liu [2], but these authors did not consider the relatively short-range Debye-shielded interactions since they were interested in neutral plasmas that are very large when measured in Debye lengths. However, non-neutral plasmas typically are many but not thousands of Debye lengths across. We will provide a unified treatment that retains both the Debye-shielded and wave contributions to the thermal diffusivity.

Before starting the analysis, it is worth noting that heat conduction should be readily measurable in non-neutral plasmas. A careful comparison between theory and experiment for this heat transfer mechanism (the exchange of parallel energy through long-range interactions) would be of interest not only as a novel effect in kinetic theory. Rosenbluth and Liu originally considered the wave transport mechanism as a possible explanation of the anomalously large heat loss through the electron channel in tokamak plasmas. More recently, Ware has discussed the enhancement of the wave transport for a non-Maxwellian particle distribution (e.g., a high energy tail) [3]. Such distributions can be produced in a trapped non-neutral plasma. Of course, the advantage of using a non-neutral plasma with $r_c \ll \lambda_D$ for such studies is that the mechanism of interest dominates the heat transport. This Letter is intended to lay the theoretical groundwork for such studies.

In order to provide the most straightforward derivation of the heat transport, we assume $\mathbf{B}$ is constant in the z direction, and we work in slab geometry. The plasma density and temperature vary in x but not in y or z.

The average rate of change of the local plasma kinetic energy density, $Q = n(x)\langle mv^2/2\rangle$, is given by the average work done on the particles by the electric field $\mathbf{E}$: $\partial Q/\partial t = \langle \mathbf{J}\cdot\mathbf{E}\rangle$, where $\mathbf{J}$ is the plasma current density. That part of the work given by $\langle \mathbf{J}_\perp \cdot \mathbf{E}_\perp\rangle$ yields the classical result for the heat flux, which we can neglect in the regime of interest, $\lambda_D \gg r_c$. Here we concentrate on the parallel work, $\langle J_z E_z\rangle$. This quantity can be evaluated to lowest order in (r_c/λ_D) by considering the evolution of guiding centers streaming parallel to the magnetic field. This evolution is governed by the guiding center Klimontovitch equation

$$\frac{\partial N}{\partial t} + v_z\frac{\partial N}{\partial z} + \frac{e}{m}E_z\frac{\partial N}{\partial v_z} = 0\,, \tag{2}$$

where $E_z(\mathbf{r},t)$ is related to $N(\mathbf{r},v_z,t)$ through Poisson's equation. Following the usual approach we break N into a smooth equilibrium distribution and a fluctuation, $N = f(x,v_z,t) + \delta N(\mathbf{r},v_z,t)$. Writing $J_z = e\int dv_z v_z N$ then implies

$$\frac{\partial Q}{\partial t}(x,t) = \langle J_z E_z\rangle = e\int dv_z v_z\langle \delta N\delta E_z\rangle\,. \tag{3}$$

The evolution of the fluctuations is described by linearizing Eq. (2):

$$\frac{\partial \delta N}{\partial t} + v_z\frac{\partial \delta N}{\partial z} + \frac{e}{m}\delta E_z\frac{\partial f}{\partial v_z} = 0\,. \tag{4}$$

This equation can be solved simultaneously with Poisson's equation by Fourier transforming in y and z and Laplace transforming in t, suppressing the time dependence of f as it evolves on a slow transport time scale:

$$\begin{aligned}(p + ik_z v_z)\delta\hat{N} = {}& \frac{e}{m}ik_z\delta\hat{\phi}\frac{\partial f}{\partial v_z} \\ &+ \delta N(t=0,x,k_y,k_z,v_z)\,,\end{aligned} \tag{5}$$

where $\delta\hat{N}(p,x,k_y,k_z,v_z)$ is the Fourier-Laplace transform of δN and $\delta N(t=0,x,k_y,k_z,v_z)$ is the Fourier transform of the initial condition. Poisson's equation provides a second relation between $\delta\hat{\phi}$ and $\delta\hat{N}$, and yields the following solution for $\delta\hat{\phi}$ when combined with Eq. (5):

$$\begin{aligned}\delta\hat{\phi} = {}& -4\pi e\int dx'\int dv_z\delta N(t=0,x',k_y,k_z,v_z) \\ &\times \psi(p,k_y,k_z,x,x')\,,\end{aligned} \tag{6}$$

where ψ is a Green's function satisfying

$$\left[\frac{\partial^2}{\partial x^2} - k_y^2 - k_z^2 + \frac{4\pi e^2}{m}ik_z\int \frac{dv_z\partial f/\partial v_z}{p+ik_z v_z}\right]\psi(p,k_y,k_z,x,x') = \delta(x-x')\,. \tag{7}$$

We can now evaluate the right-hand side of Eq. (3), averaging over a set of uncorrelated initial fluctuations that satisfy $\langle \delta N(t=0,\mathbf{r},v_z)\delta N(t=0,\mathbf{r}',v_z')\rangle = f(x,v_z)\delta(v_z-v_z')\delta(\mathbf{r}-\mathbf{r}')$. We then employ the Bogoliubov ansatz [4] in order to evaluate the inverse Laplace transforms, assuming that the fluctuations relax to their asymptotic form on a time scale rapid compared to the transport rate. The result is

$$\begin{aligned}\frac{\partial Q}{\partial t} = {}& \frac{(4\pi e^2)^2}{m}\int dv_z v_z\int \frac{dk_y dk_z}{(2\pi)^2}k_z^2\int dx' dv_z'\pi\delta[k_z(v_z-v_z')]\,|\psi(-ik_z v_z',k_y,k_z,x,x')|^2 \\ &\times\left\{f(x',v_z')\frac{\partial f(x,v_z)}{\partial v_z} - f(x,v_z)\frac{\partial f(x',v_z')}{\partial v_z'}\right\}.\end{aligned} \tag{8}$$

The right-hand side is merely the energy integral of the Balescu-Lenard collision operator for 1D collisions [5], except that particles can be on different field lines and the plasma is inhomogeneous in the x direction.

Equation (8) can be further simplified if we assume that all interactions are short range compared to the variation in f, implying that ψ is sharply peaked in $|x-x'|$; so that the integrand may be Taylor expanded around

$x' = x$. If we also assume f is a local Maxwellian with slowly varying temperature and density, we obtain the thermal diffusion equation $\partial Q/\partial t = \nabla \cdot (\kappa \nabla T)$. The thermal conductivity κ is

$$\kappa = \frac{e^2 n}{2\pi^2 m \bar{v}} \int d^3\bar{k} \int d\bar{u}\bar{u}^2 e^{-\bar{u}^2} \times \frac{|\bar{k}_z|\bar{k}_x^2}{|\bar{k}^2 D(\omega = k_z v_z, k)|^4}, \quad (9)$$

where $\bar{u} = v_z/\bar{v}$, $\bar{\mathbf{k}} = \mathbf{k}\lambda_D$, and $D(\omega, k) = 1 + [1 + \zeta Z(\zeta)]\bar{k}^{-2}$ is the plasma dielectric function, $\zeta = \omega/\sqrt{2}k_z\bar{v}$, and $Z(\zeta)$ is the plasma dispersion function. In this derivation we employed the identity $\int dx'(x - x')^2|\psi|^2 = 2\int dk_x k_x^2/(\pi|k^2 D|^4)$, which follows from the Fourier transform in x of Eq. (7), together with the assumption that ψ is strongly peaked in $|x - x'|$.

If we consider only particles interacting via a Debye-shielded potential, so that $D = 1 + \bar{k}^{-2}$, the integrals in Eq. (9) can be performed analytically. We obtain $\kappa = \kappa^{\text{Debye}}$ where $\kappa^{\text{Debye}} = e^2 n/(48\sqrt{\pi}\, m\bar{v})$, which is of order $n\nu_c\lambda_D^2$, as expected from the previous intuitive picture.

However, if instead the exact dielectric $D(\omega, k)$ is employed in Eq. (9), the integrand diverges. This can be observed in Fig. 1, where we plot the function $g(\bar{k})$, where $g(\bar{k}) = (\pi/2\bar{k}^3)\int d\bar{u}\bar{u}^2 e^{-\bar{u}^2}/|D|^4$ is obtained by integrating the integrand in Eq. (9) over solid angles in $\mathbf{k}$. At small $\bar{k}$, g diverges because of a near zero in D caused by lightly damped plasma waves. These waves can travel long distances across the magnetic field so interactions are no longer short range, and Eq. (9) is no longer valid. Of course, in the Debye-shielding approximation, plasma waves are neglected and $g(\bar{k})$ is not singular, taking the form $g^{\text{Debye}}(\bar{k}) = \pi^{3/2}\bar{k}^5/4(1 + \bar{k}^2)^4$. This function is also plotted in Fig. 1 for comparison.

In order to obtain a finite result for the thermal transport without resorting to an *ad hoc* Debye shielding model of the interaction, we should no longer assume that

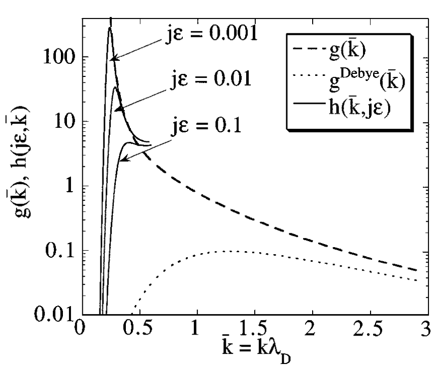

FIG. 1. The functions $g(\bar{k})$, $g^{\text{Debye}}(\bar{k})$, and $h(\bar{k}, j\epsilon)$ versus wave number $\bar{k} = k\lambda_D$; $h(\bar{k}, j\epsilon)$ is shown for three values of $j\epsilon$ where $\epsilon = \pi\lambda_D/L$. The respective areas under these curves determine the local, the Debye-shielded, and the nonlocal contributions to the thermal conductivity, i.e., κ_{local}, κ^{Debye}, and κ^j_{waves}.

the Green's function ψ in Eq. (8) is sharply peaked in $|x - x'|$. Instead, we will expand ψ in the eigenmodes of the dispersion operator appearing on the left-hand side of Eq. (7). While this can be done in general via a WKB analysis, here we will present the results of a more straightforward analysis by assuming that $n(x) = \text{const}$ between conducting plates at $x = 0$ and L, and $T(x)$ is almost constant. Working to lowest order in the variation of $T(x)$, we can neglect this variation when determining the eigenmodes, so the eigenmodes are $\sqrt{2/L}\sin k_x x$ with $k_x = n\pi/L$, and the Green's function is

$$\psi(\omega, k_y, k_z, x, x') = \frac{2}{L}\sum_{k_x}\frac{\sin(k_x x)\sin(k_x x')}{k^2 D(\omega, k)}, \quad (10)$$

where $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$. The integrals in Eq. (8) can then be performed asymptotically when the parameter $\epsilon = \pi\lambda_D/L$ is small. Since $|\psi|^2$ appears in the integrand of Eq. (8), there appears a double sum over k_x, $\sum_{k_x k_x'}$. However, the flux is dominated by $k_x \simeq k_x'$; otherwise the integral over x' phase mixes away (because the integrand is dominated by k_x of $O(\lambda_D^{-1})$, so $\sin k_x x'$ oscillates rapidly). Equation (8) then reduces to

$$\frac{dQ}{dt} = \frac{(4\pi e^2)^2}{mL^2T^2}\int dv_z v_z^2 f^2(v_z)\int\frac{dk_y dk_z}{(2\pi)^2}|k_z| \times \sum_{k_x k_x'}\frac{1}{k^2 D(\omega, k)k'^2 D^*(\omega, k')}\bigg|_{\omega = k_z v_z}\cos(\Delta k_x x) \times \int dx' \cos(\Delta k_x x')[T(x') - T(x)], \quad (11)$$

where $\Delta k_x = k_x - k_x'$, and $k' = \sqrt{k_x'^2 + k_y^2 + k_z^2}$.

For $\epsilon \to 0$ the integrand takes two asymptotic forms, depending on the size of $k\lambda_D$, i.e., on the wave number of each eigenmode mediating the interaction. These two forms can be asymptotically matched at $k\lambda_D = 0.4$. When $k\lambda_D > 0.4$ there are no lightly damped waves and $1/D$ is slowly varying in k. By Taylor expanding $1/k'^2 D(\omega, k')$ in Δk_x, one finds that the $(\Delta k_x)^0$ term vanishes because

$$\sum_{\Delta k_x}\cos\Delta k_x x\int_0^L dx'\cos\Delta k_x x'[T(x') - T(x)] = 0, \quad (12)$$

and the $(\Delta k_x)^1$ term vanishes because it is odd in Δk_x. The $O(\Delta k_x)^2$ term leads back to the local form for the conductivity, given by Eq. (9), except that the integral is limited to $k\lambda_D > 0.4$.

In the regime $k\lambda_D < 0.4$, lightly damped waves provide the main contribution to the integral. When ω nears a zero of the dielectric function, at $\omega = \omega_r(\mathbf{k}) - i\gamma(k)$, $1/D(\omega, k)D^*(\omega, k')$ becomes sharply peaked, and this peak provides the wave contribution to the flux. Assuming that $\gamma \ll \omega_r$, and integrating over the peak, one finds

$$\int\frac{dv_z}{D(\omega, k)D^*(\omega, k')}\bigg|_{\omega = k_z v_z} \simeq \frac{2\pi/|k_z|}{\partial D/\partial\omega_r\,\partial D/\partial\omega_r'} \times \frac{2\bar{\gamma} - i\Delta\omega}{4\bar{\gamma}^2 + \Delta\omega^2}, \quad (13)$$

where $\omega_r' = \omega_r(\mathbf{k}')$, $\Delta\omega = \omega_r(\mathbf{k}) - \omega_r(\mathbf{k}')$, and $\bar{\gamma} = [\gamma(k) + \gamma(k')]/2$. The imaginary part vanishes because it is odd upon interchange of k_x and k_x'. The small k "wave" contribution to the flux is evaluated by substituting Eq. (13) into Eq. (11), taking $\Delta\omega \simeq \Delta k_x \partial\omega_r/\partial k_x$, and using the small k limit of the magnetized plasma dispersion relation: $\omega_r \cong \omega_p \bar{k}_z(1 + 3\bar{k}^2/2)/\bar{k}$, $\gamma \cong -\sqrt{\pi}\,\omega_p \times e^{-1/2\bar{k}^2-3/2}/(2\sqrt{2}\,\bar{k}^4)$, $\partial D/\partial\omega_r \sim 2(k_z\omega_p/k)^2/\omega_r^3$.

Substituting Eq. (13) into Eq. (11), we note that Eq. (11) is unchanged by the addition of any function independent of Δk_x to the right-hand side of Eq. (13) [this follows from Eq. (12)]. We therefore subtract $\pi/[|k_z|\,|\partial D/\partial\omega_r|^2\gamma(k)]$, which causes the integrand of Eq. (11) to vanish when $\Delta k_x = 0$. Then upon integrating by parts once in x', taking $\Delta k_x = j\pi/L$, turning the sum over k_x into an integral, and adding in the contribution from $k\lambda_D > 0.4$, we arrive at a finite heat transport rate:

$$\frac{\partial Q}{\partial t} = \frac{\partial}{\partial x}\sum_{j=1}^{\infty}(\kappa_{\mathrm{local}} + \kappa_{\mathrm{waves}}^{j})\hat{T}_j \sin\frac{j\pi x}{L}. \tag{14}$$

Here, $\hat{T}_j \equiv (2/L)\int_0^L dx'\,\partial T/\partial x' \sin(j\pi x'/L)$ is the Fourier transform of the temperature gradient, $\kappa_{\mathrm{local}} = e^2 n/(2\pi^2 m\bar{v})\int_{0.4}^{\infty} g(\bar{k})\,d\bar{k}$ is the contribution to the thermal conductivity due to large wave number short-range interactions, and $\kappa_{\mathrm{waves}}^{j} = e^2 n/(2\pi^2 m\bar{v})\int_0^{0.4} h(\bar{k}, j\epsilon)\,d\bar{k}$ is the lightly damped wave contribution due to small wave numbers. $h(\bar{k}, j\epsilon)$ is a resonance function derived from Eqs. (11) and (13):

$$h(\bar{k}, j\epsilon) = 2\gamma\bar{k}^2\int d\Omega\,|\bar{k}_z|\bar{k}_x^2/[(j\epsilon\bar{k}_x)^2 + 4\gamma^2\bar{k}^6], \tag{15}$$

where $d\Omega$ is the element of solid angle. This function is displayed in Fig. 1. When $j\epsilon \ll 1$, $h(\bar{k}, j\epsilon) \approx g(\bar{k})$ over a region near $\bar{k} = 0.4$, so we can asymptotically match the local and wave contributions at $\bar{k} = 0.4$. However, $h(\bar{k}, j\epsilon)$ is not divergent at small $\bar{k}$, so the heat flux is now finite, and depends on the scale length of the thermal gradient L_T through the parameter $j\epsilon$ [Eq. (14) implies that $L_T = L/j\pi$, so $j\epsilon = \lambda_D/L_T$]. Performing the required integrals over $\bar{k}$ numerically we find that $\kappa_{\mathrm{local}} = 0.0975 e^2 n/m\bar{v}$ and $\kappa_{\mathrm{waves}}^{j}$ is provided in Table I for different values of $j\epsilon$. We see that $\kappa_{\mathrm{waves}}^{j} > \kappa_{\mathrm{local}}$ only for $j\epsilon \lesssim 0.02$.

Note that the conductivity κ_{local} from the rigorous eigenmode analysis is an order of magnitude larger than the value κ^{Debye} obtained with *ad hoc* Debye shielding. This is because the interaction cannot be accurately characterized by the simple Debye-shielded dielectric response $D = 1 + \bar{k}^{-2}$, even though the range over which particles interact is limited to $O(\lambda_D)$ when the wave number of the interaction satisfies $\bar{k} > 0.4$. Off-resonant plasma waves greatly increase $g(\bar{k})$ compared to $g^{\mathrm{Debye}}(\bar{k})$ (see Fig. 1). Only for $\bar{k} \gtrsim 3$ do these functions approach one another.

TABLE I. Wave contribution to heat transport.

$j\epsilon$	$\kappa_{\mathrm{waves}}^{j}/(e^2 n/m\overline{v})$
0.1	0.021
0.05	0.046
0.01	0.168
0.005	0.270
0.001	0.827
0.0005	1.367
0.0001	4.645

For $j\epsilon \to 0$, an asymptotic analysis of $h(\bar{k}, j\epsilon)$ reveals that $\kappa_{\mathrm{waves}}^{j} \simeq [2k^{*4}/(3\pi j\epsilon)]e^2 n/m\bar{v}$, where $\bar{k}^*(j\epsilon)$ is the wave number at the maximum of $h(\bar{k}, j\epsilon)$, given by $\bar{k}^* \sim 1/\sqrt{-2\ln(-j\epsilon/\ln j\epsilon)}$ for $j\epsilon \ll 1$. For example, for $j\epsilon = 0.001$, we obtain $\kappa_{\mathrm{waves}}^{j} \approx (0.2)^3 n\nu_c\lambda_D L_T$, in agreement with the heuristic estimate of Eq. (1).

In conclusion, when $\lambda_D \gg r_c$ we have shown that the classical theory of magnetized plasma thermal conductivity is not relevant. In this regime the conductivity is independent of the magnetic field strength. For thermal scale lengths $L_T \gtrsim 100\lambda_D$ across the B field, emission and absorption of lightly damped plasma waves is the dominant heat transport mechanism. For smaller scale lengths, short-range interactions on the scale of λ_D provide the dominant transport mechanism. Both the short-range and wave interaction mechanisms were obtained as limiting cases of a unified transport theory. It may be worth noting that a similar wave mechanism also enhances the cross-field like-particle collisional particle flux in plasmas which are sufficiently large [6]. Experiments to measure the thermal conductivity and particle flux in both Maxwellian and non-Maxwellian non-neutral plasmas are now under way.

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[1] M. N. Rosenbluth and A. N. Kaufmann, Phys. Rev. **109**, 1 (1958).
[2] M. N. Rosenbluth and C. S. Liu, Phys. Fluids **19**, 815 (1976).
[3] A. A. Ware, Phys. Fluids B **5**, 2769 (1993).
[4] D. C. Montgomery and D. A. Tidman, *Plasma Kinetic Theory* (McGraw-Hill, New York, 1964,) p. 66.
[5] S. Ichimaru, *Basic Principles of Plasma Physics* (W. A. Benjamin, London, 1973), p. 28.
[6] D. H. E. Dubin and T. M. O'Neil, *Proceedings of the US-Japan Joint Institute for Fusion Theory Program* (Nagoya Univ., Nagoya, Japan, 1986), pp. 265–279.