Eigenfunctions and eigenvalues of the Dougherty collision operator

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The Dougherty collision operator is a simplified Fokker-Planck collision operator that conserves particle number, momentum, and energy. In this paper, a complete set of orthogonal eigenfunctions of the linearized Dougherty operator is obtained. Five of the eigenfunctions have zero eigenvalue and correspond to the five conserved quantities (particle number, three components of momentum, and energy). The connection between the eigenfunctions and fluid modes in the limit of strong collisionality is demonstrated; in particular, the sound speed, thermal conductivity, and viscosity predicted by the Dougherty operator are identified. © *2007 American Institute of Physics*.
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I. BACKGROUND

In the kinetic theory of plasmas, the effect of collisions on the particle distribution function is treated by the Fokker-Planck collision operator of MacDonald, Rosenbluth, and Judd[1] (MRJ). This operator satisfies the usual properties expected of a good collision operator:

(a) it vanishes for any thermal equilibrium distribution function (any Maxwellian);
(b) it drives the plasma to thermal equilibrium in the long-time limit; that is, the long-time solution of the Boltzmann equation $\partial f/\partial t = C(f,f)$ is a Maxwellian (here f is the distribution for a given particle species and C is the MRJ operator);
(c) it conserves particle number, momentum, and energy.

In addition, the MRJ operator satisfies a property specific to plasmas:

(d) it accurately accounts for the dominance of small-angle scattering; i.e., it contains a velocity-space diffusion term.

However, inversion of the MRJ operator to find the distribution function is not tractable in most cases of interest. Therefore, it is desirable to find an operator that is invertible and yet preserves the important properties listed above.

This *ad hoc* approach to the collision operator as a means to analytic progress is not a new idea. For example, Bhatnagar, Gross, and Krook[2] (BGK) proposed a drastically simplified collision operator in 1957, and in 1958 Lenard and Bernstein[3] (LB) utilized a Fokker-Planck operator with constant diffusion and drag coefficients in order to study analytically the effect of collisions on plasma waves. However, each of these operators neglects at least one of the properties listed above and is incapable of predicting certain phenomena as a result. Specifically, the BGK operator, while conserving the necessary quantities, neglects the dominant role played by small-angle scattering in the collisional relaxation of the distribution function; as a result, in the limit of weak collisionality this operator fails to predict the dramatically enhanced relaxation that occurs over regions of velocity-space in which the distribution varies sharply. Conversely, the LB operator accounts for velocity-space diffusion but does not conserve momentum or energy; therefore, results obtained from the LB operator cannot match those from fluid theory in the limit of strong collisionality.

The focus of this article is a generalization of the LB operator, conceived of by Dougherty,[4] which retains each of the properties (a) through (d). The operator proposed by Dougherty is given by

$$C_D(f,f) = \nu \frac{\partial}{\partial \vec{v}} \cdot \left[\frac{T[f]}{m} \frac{\partial f}{\partial \vec{v}} + (\vec{v} - \vec{V}[f]) f \right], \tag{1}$$

where

$$\vec{V}[f] = \frac{1}{n} \int d\vec{v}\,\vec{v} f, \quad T[f] = \frac{1}{3n} \int d\vec{v}\, m(\vec{v} - \vec{V})^2 f,$$
$$n[f] = \int d\vec{v} f, \tag{2}$$

ν is a characteristic collision frequency, and m is the particle mass; this operator applies only to collisions involving a single species of particle. Unlike the LB operator, the Dougherty operator conserves all of the desired quantities and therefore can match onto fluid theory in the limit of strong collisionality. Note here that strongly collisional does not mean strongly coupled, but rather the weaker condition that the mean-free path between collisions is smaller than the spatial scale of interest (e.g., mode wavelength).

The advantage of the Dougherty operator is that it is analytically tractable. The sacrifice is that the velocity dependence of the Fokker-Planck coefficients is neglected, and therefore results are only qualitatively correct.

If the particle distribution function can be written as $f = f_0 + \delta f$, where δf is a small perturbation and f_0 is the Maxwellian characterized by density n_0, temperature T_0, and zero mean velocity, then one may write

$$C_D(f,f) \cong C_D(f_0,\delta f) + C_D(\delta f,f_0) \equiv \nu \frac{\partial}{\partial \vec{v}} \cdot \left[\frac{T_0}{m}\frac{\partial \delta f}{\partial \vec{v}} + \vec{v}\delta f + \frac{\delta T}{m}\frac{\partial f_0}{\partial \vec{v}} - \delta\vec{V} f_0 \right], \quad (3)$$

where

$$\delta T = (3n_0)^{-1} \int d\vec{v} m(v^2 - 3T_0/m)\delta f, \quad \delta\vec{V} = n_0^{-1}\int d\vec{v}\vec{v}\delta f. \quad (4)$$

The first two terms in Eq. (3) are identical to the LB operator, while the remaining terms are responsible for restoring momentum and energy conservation. Dougherty focuses on the inversion of this linearized operator to find δf. Following Chandrasekhar,[5] he constructs a Green's function for the linearized kinetic equation

$$\frac{\partial \delta f}{\partial t} + \vec{v}\cdot\frac{\partial \delta f}{\partial \vec{x}} + \frac{q}{mc}(\vec{v}\times\vec{B})\cdot\frac{\partial \delta f}{\partial \vec{v}} - \nu\frac{\partial}{\partial \vec{v}}\cdot\left[\vec{v}\delta f + \frac{T_0}{m}\frac{\partial \delta f}{\partial \vec{v}}\right] = \frac{1}{T_0}\delta\vec{E}\cdot\vec{v}f_0 + \nu\left[\frac{m\delta\vec{V}\cdot\vec{v}}{T_0} + \frac{\delta T}{T_0}\left(\frac{mv^2}{T_0} - 3\right)\right]f_0, \quad (5)$$

treating the right-hand side as a source term. Using the Green's function, it is possible to obtain an expression for δf in terms of $\delta\vec{\mathbf{V}}$ and δT, and this expression may be substituted in the definitions of these quantities, resulting in two algebraic equations for $\delta\vec{\mathbf{V}}$ and δT. These equations can then be solved and δf determined.

A different method for inverting the linearized Dougherty operator was introduced by DeSouza-Machado *et al.*[6] These authors expand the velocity dependence of δf in an infinite series of orthogonal basis functions (Hermite polynomials), converting the Dougherty operator to an infinite matrix acting on the vector of coefficients in the orthogonal function expansion. The Hermite polynomials diagonalize the LB part of the Dougherty operator [the first two terms in Eq. (3)], but not the whole operator.

In contrast, we expand δf in orthogonal basis functions that diagonalize the whole Dougherty operator. Most of these eigenfunctions are just the Hermite polynomials, but a few are modified by the third and fourth terms in the linearized Dougherty operator [Eq. (3)]. Physically, the modified eigenfunctions (and eigenvalues) are a consequence of the conservation properties of the Dougherty operator. One should note that the eigenfunctions diagonalize only the collision operator, not the streaming and force terms in the Boltzmann equation. These terms couple the eigenfunctions.

We note that Ng, Bhattacharjee, and Skiff[7] have determined a complete set of eigenfunctions that simultaneously diagonalize the LB operator plus streaming and force terms, in analogy with the Case–Van Kampen modes of the Vlasov equation.[8,9] An analogous theory for the Dougherty operator is not considered in this paper.

Five of the eigenfunctions of the Dougherty operator have eigenvalue zero (corresponding to conservation of particle number, three components of momentum, and energy), and these eigenfunctions are crucial in connecting onto fluid theory. We discuss the relation between these special eigenfunctions and the usual hydrodynamic modes in the limit of strong collisionality, identifying the sound speed, thermal conductivity, and viscosity predicted by the Dougherty operator.

More formally, the hydrodynamic modes arise because the streaming term in the Boltzmann equation [i.e., $ikv_z\delta f$ for $\delta f \sim \exp(ikz)$] is not diagonalized by the new eigenfunctions. Although the streaming term may be treated as a small perturbation in the limit of strong collisionality (hydrodynamic limit), it thoroughly mixes the degenerate eigenfunctions with eigenvalue zero, yielding the hydrodynamic mode eigenfunctions. These hydrodynamic eigenfunctions diagonalize both the collision operator and the streaming term in the important subspace of undamped modes. In second-order perturbation theory, the hydrodynamic modes pick up weak damping due to weak coupling to eigenfunctions outside the subspace.

II. EIGENFUNCTIONS OF THE LINEARIZED DOUGHERTY OPERATOR

We may put the linearized Dougherty operator in self-adjoint form by writing $\delta f = f_0\phi$ and substituting this expression in Eq. (3). The result is

$$C_D(f_0,\delta f) + C_D(\delta f,f_0) = f_0\nu\left[\frac{\partial^2\phi}{\partial u^2} - \vec{u}\cdot\frac{\partial\phi}{\partial\vec{u}} + \frac{\delta T}{T_0}(u^2-3) + \frac{\delta\vec{V}}{\sqrt{T_0/m}}\cdot\vec{u}\right] \equiv f_0\chi(\phi), \quad (6)$$

where we have introduced the scaled velocity $\vec{u} \equiv \vec{v}/\sqrt{T_0/m}$; the operator χ is self-adjoint with weight function f_0. In order to find the eigenfunctions of this operator, we break it into two parts—a differential operator,

$$\chi_1(\phi) \equiv \nu\left[\frac{\partial^2\phi}{\partial u^2} - \vec{u}\cdot\frac{\partial\phi}{\partial\vec{u}}\right], \quad (7)$$

and an integral operator,

$$\chi_2(\phi) \equiv \nu\left[\frac{\delta T}{T_0}(u^2-3) + \frac{\delta\vec{V}}{\sqrt{T_0/m}}\cdot\vec{u}\right]. \quad (8)$$

As mentioned above, the eigenfunctions $\psi_{n_1n_2n_3}$ of χ_1 are the products of modified Hermite polynomials;[10] that is,

$$\psi_{n_1n_2n_3} = \frac{\mathrm{He}_{n_1}(u_x)\mathrm{He}_{n_2}(u_y)\mathrm{He}_{n_3}(u_z)}{\sqrt{n_1!n_2!n_3!}}. \quad (9)$$

The corresponding eigenvalues are

$$\alpha_{n_1n_2n_3} = -\nu(n_1+n_2+n_3), \quad (10)$$

where n_1, n_2, and n_3 are non-negative integers. The functions $\psi_{n_1n_2n_3}$ satisfy the orthogonality relation

$$n_0\delta_{n_1,m_1}\delta_{n_2,m_2}\delta_{n_3,m_3} = \int d\vec{u}\psi_{n_1n_2n_3}f_0\psi_{m_1m_2m_3} \equiv n_0\langle\psi_{n_1n_2n_3}|\psi_{m_1m_2m_3}\rangle. \quad (11)$$

We observe that any $\psi_{n_1n_2n_3}$ that satisfies $\chi_2(\psi_{n_1n_2n_3})=0$ is an eigenfunction of the total operator χ with eigenvalue $\alpha_{n_1n_2n_3}$. We therefore express χ_2 in terms of inner products with the functions $\psi_{n_1n_2n_3}$:

$$\chi_2(\phi) = \nu\Big[\tfrac{2}{3}\langle\psi_{200}+\psi_{020}+\psi_{002}|\phi\rangle(\psi_{200}+\psi_{020}+\psi_{002}) + \langle\psi_{100}|\phi\rangle\psi_{100} + \langle\psi_{010}|\phi\rangle\psi_{010} + \langle\psi_{001}|\phi\rangle\psi_{001}\Big]. \quad (12)$$

Evidently, $\chi_2(\phi)$ is the projection of ϕ onto $\psi_{200}+\psi_{020}+\psi_{002}$, ψ_{100}, ψ_{010}, and ψ_{001}. Therefore, for almost every $\psi_{n_1n_2n_3}$, $\chi_2(\psi_{n_1n_2n_3})=0$, and in each such case, $\psi_{n_1n_2n_3}$ is an eigenfunction of χ with eigenvalue $\alpha_{n_1n_2n_3}$. Hereafter, we refer to these eigenfunctions and eigenvalues of χ as $\phi_{n_1n_2n_3}$ and $\lambda_{n_1n_2n_3}$, respectively. The exceptions, for which the projection in Eq. (12) is nonzero, are clearly ψ_{200}, ψ_{020}, ψ_{002}, ψ_{100}, ψ_{010}, and ψ_{001}. It is straightforward to find six additional eigenfunctions of χ to replace these exceptions. A sensible choice is

$$\phi_{100} \equiv u_x, \quad \phi_{010} \equiv u_y, \quad \phi_{001} \equiv u_z, \quad \phi_{200} \equiv \frac{1}{\sqrt{6}}(u^2-3), \quad (13)$$

with eigenvalues $\lambda_{100}=\lambda_{010}=\lambda_{001}=\lambda_{200}=0$, and

$$\phi_{020} \equiv \frac{1}{\sqrt{3}}\left[u_z^2 - \frac{1}{2}(u_x^2+u_y^2)\right], \quad \phi_{002} \equiv \frac{1}{2}(u_x^2-u_y^2), \quad (14)$$

with eigenvalues $\lambda_{020}=\lambda_{002}=-2\nu$. Defined in this manner, the eigenfunctions ϕ_{000}, ϕ_{100}, ϕ_{010}, ϕ_{001}, and ϕ_{200}, which span the null-space of χ, correspond to particle number, x, y, and z momentum, and kinetic energy. These eigenfunctions also satisfy the orthogonality relation given by Eq. (11).

In Refs. 7–9, proofs of completeness are nontrivial, since in each case the eigenmodes are constructed "from scratch" and not from a set of special functions whose completeness has already been established. In contrast, the completeness of the eigenfunctions found above follows immediately from the completeness of the modified Hermite polynomials. Except for ϕ_{200}, ϕ_{020}, and ϕ_{002}, the eigenfunctions $\phi_{n_1n_2n_3}$ are given by Eq. (9).

$$\phi_{n_1n_2n_3} \equiv \psi_{n_1n_2n_3},$$

where the functions $\psi_{n_1n_2n_3}$ are defined by Eq. (9). The three exceptions, i.e., ϕ_{200}, ϕ_{020}, and ϕ_{002}, are mutually orthogonal, and each can be expressed as a linear combination of the functions ψ_{200}, ψ_{020}, and ψ_{002}. Therefore, the eigenfunctions $\phi_{n_1n_2n_3}$ span the same space as do the functions $\psi_{n_1n_2n_3}$. Since the set $\{\psi_{n_1n_2n_3}\}$ is known to be complete, it follows that the set $\{\phi_{n_1n_2n_3}\}$ must be complete as well.

As a simple demonstration of the utility (and basic consequences) of this complete set of eigenfunctions, we consider the linearized kinetic equation

$$\frac{\partial\delta f}{\partial t} = C_D(f_0,\delta f) + C_D(\delta f, f_0) \quad (15)$$

which governs the evolution of a small, spatially uniform perturbation in the distribution. The solution can be written down immediately in terms of the eigenfunctions found above,

$$\delta f(\vec{u},t) = f_0(u)\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty} a_{n_1n_2n_3}\phi_{n_1n_2n_3}(\vec{u})\exp[\lambda_{n_1n_2n_3}t], \quad (16)$$

where the coefficients $a_{n_1n_2n_3}$ are determined from $\delta f(\vec{u},t=0)$. Note that all of the eigenvalues $\lambda_{n_1n_2n_3}$ are negative except for λ_{000}, λ_{100}, λ_{010}, λ_{001}, and λ_{200}, which are zero. Thus, the initial perturbations in density, fluid-velocity, and internal energy—δn, $\delta\vec{V}$, and δT, respectively—are preserved; all other components of the initial perturbation relax on a timescale ν^{-1} or faster. In other words, we find that

$$\lim_{t\to\infty} f = \frac{n_0}{(2\pi T_0/m)^{3/2}}e^{-u^2/2}\left[1+\frac{\delta n}{n_0}+\frac{\delta\vec{V}\cdot\vec{u}}{\sqrt{T_0/m}}+\frac{\delta T(u^2-3)}{T_0}\right]. \quad (17)$$

Since $\delta n/n_0$, $|\delta\vec{V}/\sqrt{T_0/m}|$, and $\delta T/T_0$ are small in comparison to unity, this time-asymptotic expression is equivalent to a Maxwellian with density $n_0+\delta n$, mean velocity $\delta\vec{V}$, and temperature $T_0+\delta T$.

In certain circumstances (for example, if the plasma of interest is magnetized), it may be useful to work in cylindrical velocity coordinates, which we define by

$$u_\perp = \sqrt{u_x^2+u_y^2}, \quad \theta_u = \tan^{-1}(u_y/u_x). \quad (18)$$

In these coordinates, the $u_\perp$ dependence of the eigenfunctions of χ may be expressed in terms of the associated Laguerre polynomials $L_n^m(x)$. Specifically, the functions

$$\phi^{[1]}_{n_\perp n_z m_1} = u_\perp^{|m_1|}L_{n_r}^{|m_1|}(u_\perp^2/2)\sin(m_1\theta_u)\mathrm{He}_{n_z}(u_z),$$
$$\phi^{[2]}_{n_\perp n_z m_2} = u_\perp^{|m_2|}L_{n_r}^{|m_2|}(u_\perp^2/2)\cos(m_2\theta_u)\mathrm{He}_{n_z}(u_z) \quad (19)$$

are eigenfunctions of χ with eigenvalues

$$\lambda^{[1]}_{n_r n_z m_1} = -\nu(n_z+2n_r+|m_1|), \quad \lambda^{[2]}_{n_r n_z m_2} = -\nu(n_z+2n_r+|m_2|), \quad (20)$$

provided that $\{n_\perp,n_z,m_2\}\neq\{1,0,0\}$, $\{0,1,0\}$, $\{0,0,1\}$, $\{0,2,0\}$, $\{0,0,2\}$, and that $\{n_\perp,n_z,m_1\}\neq\{0,0,1\}$; here, $n_\perp, n_z$, and m_2 are non-negative integers and m_1 is a positive integer. The remaining eigenfunctions are

$$\phi^{[1]}_{001} = u_\perp\sin\theta_u, \quad \phi^{[2]}_{001} = u_\perp\cos\theta_u,$$
$$\phi^{[2]}_{010} = u_z, \quad \phi^{[2]}_{100} = u_\perp^2+u_z^2-3, \quad (21)$$

with eigenvalues $\lambda^{[1]}_{001}$, $\lambda^{[2]}_{001}$, $\lambda^{[2]}_{010}$, $\lambda^{[2]}_{100}=0$ and

$$\phi^{[2]}_{002} = u_\perp^2\cos(2\theta_u), \quad \phi^{[2]}_{020} = 2u_z^2-u_\perp^2, \quad (22)$$

with eigenvalues $\lambda^{[2]}_{002}$, $\lambda^{[2]}_{020}=-2\nu$.

III. THE LIMIT OF STRONG COLLISIONALITY

If a collision operator is to remain an accurate model when the effect of collisions becomes strong, it must conserve particle number, momentum, and energy. The reason is that the fluid description of the plasma in this limit is characterized by hydrodynamic modes that decay slowly compared with the typical collisional relaxation time, and the existence of such modes requires that these quantities be conserved. Because the Dougherty operator respects these conservation laws, it naturally gives rise to fluid-like behavior when collisions are strong.

To see that the Dougherty operator gives rise to such a fluid theory, we imagine a single-species plasma and consider how a perturbation of the form

$$\delta f(\vec{u},z,t) = f_0(u)\phi(\vec{u},t)e^{ikz} \tag{23}$$

evolves according to this operator.[11] Neglecting external and mean field forces, the linearized kinetic equation corresponding to the Dougherty operator is

$$[iku_z\sqrt{T_0/m} - \chi]\phi = -\frac{\partial\phi}{\partial t}. \tag{24}$$

Solving this equation is equivalent to finding the eigenfunctions of the operator $K \equiv iku_z\sqrt{T_0/m} - \chi$. If collisions are sufficiently strong (i.e., $\nu \gg k\sqrt{T_0/m}$), then $iku_z\sqrt{T_0/m}$ may be treated as a perturbation to χ in Eq. (24). Thus, if $\Phi_{n_1n_2n_3}$ and $\Lambda_{n_1n_2n_3}$ are the eigenfunctions and eigenvalues of K, then as a first approximation,

$$\Phi_{n_1n_2n_3} = \phi_{n_1n_2n_3}, \quad \Lambda_{n_1n_2n_3} = -\lambda_{n_1n_2n_3}, \tag{25}$$

provided that $\lambda_{n_1n_2n_3}$ is nondegenerate.

However, in the degenerate subspace for which $\lambda_{n_1n_2n_3} = 0$, one must diagonalize the perturbation, $iku_z\sqrt{T_0/m}$, in order to obtain the correct lowest-order approximation to the eigenfunctions of K. In this degenerate subspace, in the basis $\{1,\ u_x,\ u_y,\ u_z,\ (u^2-3)/\sqrt{6}\}$, the operator $iku_z\sqrt{T_0/m}$ has the following matrix representation:

$$iku_z\sqrt{T_0/m} = ik\sqrt{T_0/m}\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \sqrt{2/3} \\ 0 & 0 & 0 & \sqrt{2/3} & 0 \end{bmatrix}. \tag{26}$$

The eigenvectors and eigenvalues of this "degenerate block" are

$$\begin{aligned} &\Phi^{(0)}_{000} = \frac{1}{\sqrt{5}}\left[-\sqrt{2} + \frac{1}{\sqrt{2}}(u^2-3)\right], \quad \Lambda^{(1)}_{000} = 0, \\ &\Phi^{(0)}_{100} = u_x, \quad \Lambda^{(1)}_{100} = 0, \quad \Phi^{(0)}_{010} = u_y, \quad \Lambda^{(1)}_{010} = 0, \\ &\Phi^{(0)}_{00\pm1} = \sqrt{\frac{3}{10}}\left[1 \pm \sqrt{\frac{5}{3}}u_z + \frac{1}{3}(u^2-3)\right], \\ &\Lambda^{(1)}_{00\pm1} = \pm ik\sqrt{\frac{5T_0}{3m}}. \end{aligned} \tag{27}$$

The second-order corrections to these eigenvalues are given by the formula

$$\Lambda^{(2)}_{n_1n_2n_3} = \sum_{\substack{n_1',n_2',n_3' \\ s\cdot t\cdot\Lambda^{(0)}\neq 0}} \frac{|(\Phi^{(0)}_{n_1n_2n_3}, ikv_z\Phi^{(0)}_{n_1'n_2'n_3'})|^2}{-\Lambda^{(0)}_{n_1'n_2'n_3'}}; \tag{28}$$

they are $\Lambda^{(2)}_{000} = k^2T_0/3\nu m$, $\Lambda^{(2)}_{100} = \Lambda^{(2)}_{010} = k^2T_0/2\nu m$, and $\Lambda^{(2)}_{00\pm1} = 4k^2T_0/9\nu m$. Since Λ_{000}, Λ_{100}, Λ_{010}, and $\Lambda_{00\pm1}$ are smaller than all other eigenvalues of K by at least a factor of $k\sqrt{T_0/m}/\nu$, the time-asymptotic behavior of δf is dictated by Φ_{000}, Φ_{100}, Φ_{010}, and $\Phi_{00\pm1}$. Specifically, for sufficiently large time t (i.e., $\nu t \gg 1$), a hydrodynamic phase ensues, during which δf is given by

$$\begin{aligned} \delta f \cong f_0e^{ikz}\Big[&A_{000}\Phi_{000}e^{-k^2T_0t/3\nu m} + A_{100}\Phi_{100}e^{-k^2T_0t/2\nu m} \\ &+ A_{100}\Phi_{100}e^{-k^2T_0t/2\nu m} \\ &+ A_{001}\Phi_{001}e^{-4k^2T_0t/9\nu m - ik\sqrt{5T_0/3m}t} \\ &+ A_{00-1}\Phi_{00-1}e^{-4k^2T_0t/9\nu m + ik\sqrt{5T_0/3m}t}\Big], \end{aligned} \tag{29}$$

where the coefficients A_{000}, A_{100}, A_{010}, and $A_{00\pm1}$ are determined from $\phi(\vec{u},t=0)$.

The first term on the right-hand side of Eq. (29) is properly identified as a heat conduction mode; the second and third terms represent viscous relaxation; the fourth and fifth terms are counterpropagating, damped sound waves.

The eigenvalues Λ_{000}, Λ_{100}, Λ_{010}, and $\Lambda_{00\pm1}$ (corresponding to heat conduction, viscous relaxation, and sound modes) can be compared with the corresponding eigenvalues of the linearized hydrodynamic equations for the plasma (neglecting external and mean-field forces). This comparison provides a means by which to obtain the viscosity μ, thermal conductivity K, and second viscosity ζ, that result from the Dougherty collision operator. In this manner, we find that

$$\mu = \frac{1}{2}n_0\frac{T_0}{\nu}, \tag{30}$$

$$K = \frac{5}{6}n_0\frac{T_0}{m\nu}, \tag{31}$$

and $\zeta = 0$.

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