Re-Constructing Chaotic Vortex Trajectories from Confined Plasma Data

J. B. Kadtke[1], T. Mitchell[2], C. F. Driscoll[2], and K. S. Fine[2]

[1] *Institute for Pure and Applied Physical Sciences,*
University of California at San Diego,
IPAPS-0075, 9500 Gilman Dr., La Jolla, Ca. 92093, USA
[2] *Physics Department,*
University of California at San Diego,
IPAPS-0075, 9500 Gilman Dr., La Jolla, Ca. 92093, USA

Abstract

The dynamics of magnetically confined electron columns is known to be isomorphic to the dynamics of fluid vorticity obeying the 2-D Euler Equation, and experiments studying these properties have been conducted at UCSD for several years. One goal is to compare the experimentally observed plasma dynamics to theoretical predictions about the long-time behaviour of Hamiltonian point vortex models. To do so, it is necessary to extract detailed physical space trajectories for the isolated electron 'vortices'. Here, we present a preliminary method which allows the re-construction of the physical trajectories of the centers-of-vorticity of the electron column, from measured electrodynamic image charges on the walls of the conducting containment vessel. These will then be compared to the dynamics of point vortex model in circular boundaries.

1 Introduction

In this brief paper, we will describe a current experimental and theoretical effort at UCSD to deduce the dynamics of line columns of electron plasmas from voltage signals on probes in the containment vessel. The point of this exercise will be to compare this dynamics to the chaotic motion of fluid point vortices obeying the 2-D Euler Equation, and to experimentally validify certain theoretical predictions. The connection between these two seemingly disparate fields is not obvious, and will be explained to some detail below. As motivation for this discussion, however, we will first state that these experiments represent one of the best experimental realizations of 2-D inviscid fluid dynamics yet known, where the fluid dynamics is represented by the dynamics of the continuous electron distribution and is nearly exact. To illustrate this, it has been found that the effects of viscocity are negligible even for 10^4 to 10^5 characteristic time units in this system, well beyond the ability of many numerical simulations to faithfully reproduce the time evolution due to error propogation.

These experiments, conducted by Fred Driscoll and associates, therefore represent a remarkable new tool to investigate phenomena in a variety of fields[1]. For the plasma community, questions about transport and confinement in contained non-neutral plasmas can be addressed, such as predictions about eigenmode instability. For the fluid dynamics community, this system represents an experiment which can study the behaviour of 2D inviscid fluid structures on long time scales. Two-dimensional inviscid fluid systems have taken on great importance in the last decade or so, and headway has been made recently in understanding 2-D turbulence. A particular problem which we shall be interested in here is the dynamics of a few strong rotational singularities, or vortices. Much theoretical work has been done on this old system in recent years which describes the chaotic dynamics of these systems[2,3]. Our eventual aim described here will be to experimentally observe the behaviour of chaotic vortex systems, and to compare with theory.

A range of technical problems exist in extracting would-be vortex trajectories from the evolution of a distribution of electrons in a containment vessel. At present, aspects of the analysis are still being

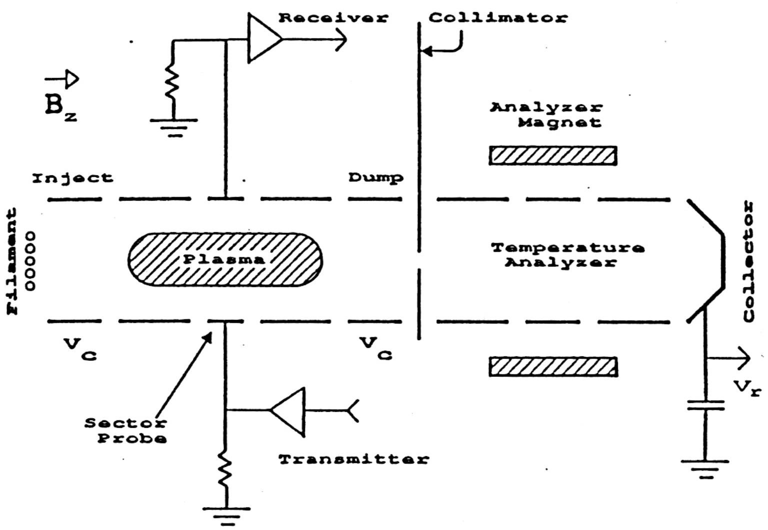

Figure 1: Schematic of experimental plasma apparatus at UCSD, called 'EV'. This machine has recently been replaced by 'CamV', which subsitutes a CCD camera for the 'collector' above, and which can make instantaneous plasma density maps.

developed, and the latest generation of the apparatus has only recently been completed. As such, most of the results presented here are preliminary, however they suggest some fascinating possibilities. The remainder of this paper will discuss these technical problems, show results, and indicate the directions to be taken in the near future.

2 Vortex Dynamics from Electron Columns

We consider a physical system consisting of a cylindrical conducting containment vessel with a magnetic field **B** aligned along the cylinder axis. A representation of the actual experimental configuration at UCSD is shown in Figure 1. Electrons injected into this vessel perform cyclotron motion whose gyro-radius can be made exceptionally small by choosing **B** large, and the electron motion is well described by the guiding center approximation. By adding negative voltages to the ends, the electrons are contained axially within the cylinder, and for sufficient kinetic energies their end-to-end bounce motion averages over any 3-D variations, allowing a 2-D $(\mathbf{r}, \theta)$ description. These columns in turn induce an electrostatic image charge on the walls of the conducting containment vessel, giving rise to large electric fields **E** and generating $\mathbf{E} \times \mathbf{B}$ drift velocities which drive the dynamics. Over 25 years ago, Levy showed analytically that the resulting motion for this system is exactly isomorphic to that produced by the 2D Euler Equations[4]. The correspondence (see Ref. 4) yields a 'vorticity' Ω expressed as $\Omega = \nabla \times \mathbf{V} = n(4\pi ec/B)\hat{\mathbf{z}}$, where $\mathbf{V} = -(c/B)\nabla\phi \times \hat{\mathbf{z}}$, and the potential ϕ is given by the Poisson equation $\nabla^2\phi = 4\pi en$. Thus, a distribution of electrons with density $n(\mathbf{r}, \theta)$ evolves exactly as a distribution of vorticity $\Omega(\mathbf{r}, \theta)$ would in a uniform 2D inviscid, incompressible fluid, such as water.

The experimental plasma system does have some important advantages over actual fluid experiments, however. Firstly, the internal viscocity of the electron flow is extremely low, with viscous effects becoming significant only for time scales which are 10^5 times longer than the typical dynamical time scales. Secondly, because of the strong axial magnetic field the dynamics remains essentially two-dimensional, a property very difficult to achieve in real fluids. Thirdly, because there are typically no electrons near the walls and hence no shear forces from wall interactions, the boundary conditions are in practice free-slip. Because one or another of these effects have been problematic to all fluid experiments in the past, the plasma system represents an important development.

For fluid dynamicists, the plasma system represents an important opportunity to compare the dynamics of real, extended vortices to that of 2-D, inviscid Hamiltonian point vortices. Here, the spatially-extended electron columns can be approximated as point vortices located at the distributions' center-of-vorticity. Research on point vortex systems has been quite active in the last decade, primarily devoted to long-time dynamical properties and the onset and statistics of chaotic motion. Because point vortex systems are also finding many applications to real physical systems, it is important to verify and study many of the theoretical predictions. The primary approach for the theoretical and numerical analyses of chaotic motion is to study the global properties of the long-time dynamics in the phase space of the dynamical system. Phase space structures such as elliptic (stable) and hyperbolic (unstable) fixed points, separatrices, etc. play a vital role in the topology of the physical flow space of vortex systems, determining global flow properties. Some experimental work at UCSD has already examined some of these properties for the two-vortex Havelock system[5]. Resolution of these structures, however, requires that individual trajectories in the phase space can be followed for periods which are long on the dynamical time scales. Consequently, it is also necessary that detailed information about the trajectories of individual electron columns in the observed experimental plasma be resolved for long times. The extraction of vortex trajectories from the evolving plasma remains a current and non-trivial problem, and our attempt using wall signals will be described in Section 4.

3 Experimental Apparatus

The electron columns are contained in a grounded conducting cylinder with a uniform axial magnetic field of up to 20 KGauss. Electrons are injected into the cylinder by ohmically heating a tungsten filament, and then trapped by applying negative voltages on the ends of the cylinder. Desired initial conditions can be created using various plasma manipulation methods, and containment times can be hundreds of seconds. Local variations in plasma densities are typically less than 0.1% from one plasma injection to the next. Typical operating parameters include a plasma column length of $L_p \sim 30$ cm, a plasma radius of $R_p \sim$ 1-2 cm, a radial electric field of $E \sim -7$ V/cm, and local electron number densities of $n \sim 5 \times 10^6$ cm^{-3}. The characteristic thermal energy of the electrons is typically $KT \sim 1$ eV, the cyclotron radius $r_c \sim 60$ μm, an $\mathbf{E} \times \mathbf{B}$ rotation frequency of $f_e \sim 140$ kHz, and end-to-end bounce rates f_b satisy $f_b \sim$ 10-50 f_e.

To date, the primary diagnostic tool for the experiments has been the construction of $(\mathbf{r}, \theta)$ plasma density maps which are obtained by 'dumping' the plasma out one end of the cylinder at a particular time, and recording the light intensity generated by the electrons incident on a phosphor screen. An example of an single-time density map is given in Figure 2, showing an evolved 'two-vortex' state. While this method gives excellent resolution for instantaneous plasma density maps, little detailed information can be obtained about the continuous time evolution of the plasma columns, which is necessary for a dynamical systems analysis.

An alternate approach now being developed, which may yield long-time evolution information, is the utilization of 'sector probe' signals of the vortex image charges from the containing vessel. The sector probes are electrically isolated conducting sections of the containment vessel, which are arranged in axially symmetric rings, and contain up to eight individual isolated sectors per ring. These sectors act as antennas, producing voltage signals which are reflect the strength and location of the electron columns vessel to high sensitivity. Eight or more signals can be recorded simultaneously, with up

Figure 2: Plasma electron density map of a two-vortex system, made with the EV machine. Density contours increase in steps of 7.3×10^5 cm^{-3}. Partial arcs represent the cylinder walls. Much of the density irregularities are due to measurement discretization, which are greatly reduced using the new CamV machine.

to 12 bit voltage accuracy, and recordings can be maintained for 0.1 seconds or more at sampling rates of 10^5/sec. The frequency spectrum of the wall signals give some information about the column dynamics, however detailed positional information is not immediate. In the next section we describe a method to extract the latter.

4 Analysis of Sector Probe Data

The important connection between the sector probe signals and the positions of the electron columns can be made through the Kapetanakos' relation[6]. This analytic relation gives the image charge Q_{sect} induced on a sector of angular width $\Delta\theta$ by a circular cross-section column of electrons at $(\mathbf{r},\theta)$ with linear electron density N_L, as an expansion in angular harmonic modes:

$$Q_{sect} = N_L e\Delta\theta/2\pi + (2N_L e\Delta\theta/\pi)\sum_{n=1}^{\infty}\sin(n\Delta\theta/2)\cos(n\theta)(r^n/n) \qquad (1)$$

where θ is the relative angle between the column and the center of the given sector. The signal on a given sector for two or more columns is just the sum of the individual column contributions. The final relationship to the voltages measured as sector signals is obtained by writing a conversion factor for the sensing circuit, i.e. $V_{sect} = G_{amp}Q_{sect}$, where G_{amp} represents the net total scaling constant due to amplification, capacitative coupling, etc. of the circuit.

Using this relationship, it is possible to define an 'inverse problem' for our system[7], in the following sense: given j measured sector signals V_j, what are the k pairs of $(\mathbf{r}_k,\theta_k)$ for a k-vortex system that could produce the observed V_j? This inverse problem can easily be re-written as a nonlinear least-squares problem: given the angular position and width $\Delta\theta$ of each sector, one can write an equation from (1) for each sector (and hence each sector signal $V_{sect} = G_{amp}Q_{sect}$), which is set equal to the observed signals V_j. The $(\mathbf{r}_k,\theta_k)$ are left as unknowns, to be solved for at each time step. A cost function is constructed consisting of the sum of squares of differences between the predicted sector equation values from (1), and the observed values V_j. We therefore construct a system of j equations in $2k$ unknowns, solvable as long as $j \geq 2k$. In practice, we utilize the nonlinear least-squares fitting package DNLS1, available from Linpack, to extract the $(\mathbf{r}_k,\theta_k)$ at each time step, with the V_j being available as digitized computer files.

Although conceptually simple, in practice we have encountered a number of significant technical problems with this method. Firstly, significant filtering of the V_j must be done to remove spurious low

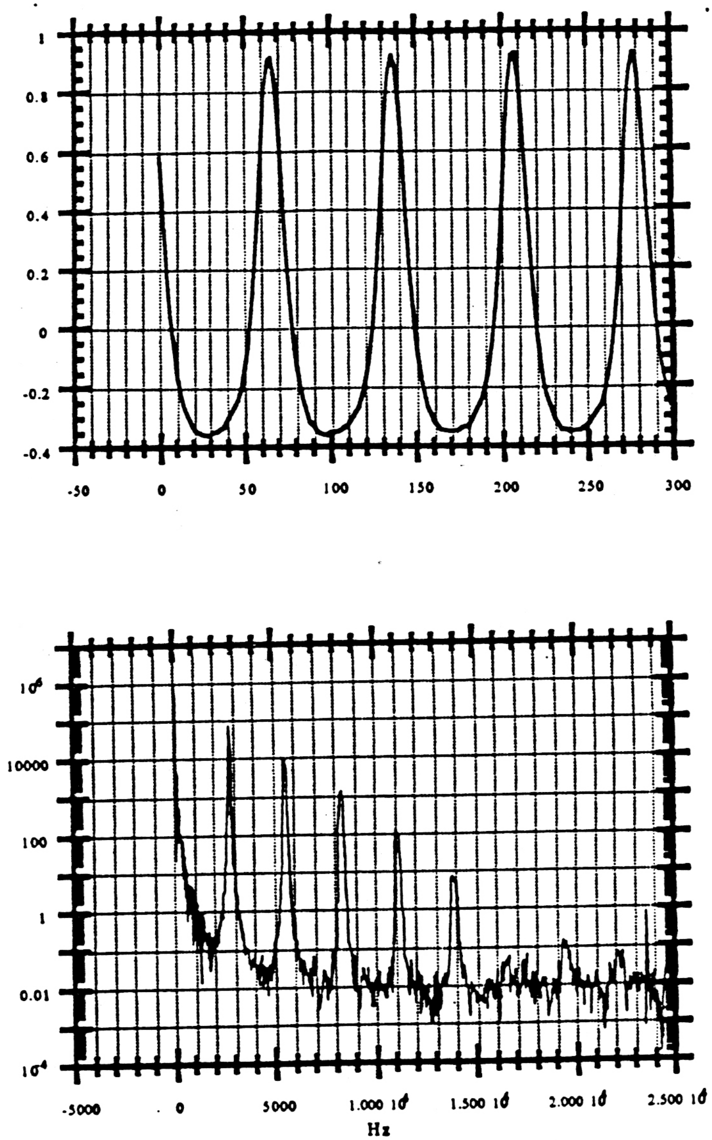

Figure 3: Plots showing an actual sector signal (top) from a one-vortex system, and the corresponding power spectrum (bottom). Top plot is of normalized voltage amplitude vrs. time (410 microseconds). Signal is plotted before any filtering occurs. Power spectrum shows extremely low noise floor ($>$ 60 dB SNR), and low frequency contamination. Signals are typically high-pass filtered using a cut-off of $\sim$ 2000 Hz.

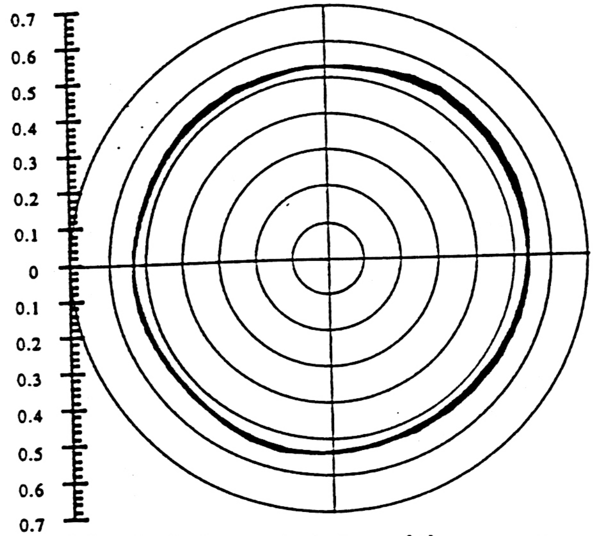

Figure 4: Polar plot of the physical-space trajectory of the one-vortex system measured from the EV apparatus, reconstructed from sector signals. Ideal motion should be a perfect circle with radius $\sim$ 0.54. Small amount of scatter is due to noise, symmetry effects, and calibration errors, which should be minimized with the new CamV apparatus.

frequency noise components, and to correct for phase distortions induced by the amplifier chain. Secondly, care must be taken to calculate the scaling constants G_{amp}, and single vortex 'calibration runs' are done to accomplish this. Thirdly, an interesting problem arises because of the symmetric positioning of the sector probes, and the insensitivity of the Kapetanakos relation to rotational direction. These effects produce a degeneracy in the possible position solutions (r_k, θ_k), at certain values of θ_k, often making it impossible to calculate continuous trajectories. Because of this effect, and the limitation to four sector probes on the previous generation of experimental apparatus, it has to date proven impossible to extract trajectories for 2 or more individual electron columns, which are of prime interest. Results from the latest apparatus, which can utilize up to 12 sector probe signals, will be available shortly.

5 Results and Conclusions

To date, the extraction of trajectories for single vortex systems has been successfully demonstrated. Figure 3 shows an example of an observed sector probe signal for a single vortex system, and also shows the power spectral density (PSD) for the signal. It should be noted that the in-band signal-to-noise ratio is quite high, typically being 60 dB or higher. The four sector probe signals for this experimental run were then utilized to solve for the single vortex position (r, θ) numerically, using the nonlinear least-squares method described above. The results of this calculation are shown in Figure 4, showing the reconstructed physical space trajectory of the vortex (electron column). Slight inaccuracies still exist due to the symmetry and scaling problems mentioned previously. Given the exact trajectory information obtained in this way, it should be possible to calculate Poincare sections (in this case trivial), examine the vicinity of elliptic and hyperbolic fixed points, and otherwise perform the relevant global phase space analysis appropriate for a Hamiltonian system.

With the recent availability of significantly improved experimental equipment, providing up to twelve sector probe signals, it will now be possible to analyze two or even three vortex systems. This will make possible the study of a variety of theoretical predictions about the dynamical properties of few-vortex systems, including the onset of chaotic motion and long term statistical properties. A primary goal will be to examine the onset of chaos via the generation of stochastic layers in the phase space representation. These experiments can also be used to check the validity of the electrodynamic-Eulerian fluid analogy, and to observe the effects of finite core size on the point vortex predictions. Finally, these experiments will of course provide valuable insights into evolution and transport properties of plasma systems as well. We hope to present these results in the near future.

Acknowledgements

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