Two Experimental Regimes of Asymmetry-Induced Transport

Jason M. Kriesel and C. Fred Driscoll

Physics Dept., University of California, San Diego CA 92093-0319

Abstract. In a cylindrical trap, azimuthally asymmetric electric or magnetic fields (such as inherent trap asymmetries) cause the cross-magnetic-field transport of particles, leading to bulk radial expansion and eventually to particle loss at the trap walls. Experiments with applied electrostatic asymmetries identify two different transport regimes, "slightly-rigid" and "highly-rigid". Here the plasma rigidity, $\mathcal{R} \equiv \overline{f_b}/f_E$, is the ratio of the axial bounce frequency to the azimuthal $\mathbf{E} \times \mathbf{B}$ rotation frequency. In the slightly-rigid regime ($1 < \mathcal{R} < 10$), the transport scales as $V_a\,\mathcal{R}^{-2}$, where V_a is the applied asymmetry strength. This $\mathcal{R}^{-2} \propto L^2/B^2$ scaling has previously been observed for transport due to inherent trap asymmetries. The "$V_a\,\mathcal{R}^{-2}$" mechanism appears to "turn-off" as the rigidity is increased into the range $\mathcal{R} \gtrsim 10$. In the highly-rigid regime ($\mathcal{R} > 20$), the transport scales as $V_a^2\,\mathcal{R}^0$.

INTRODUCTION

Non-neutral plasmas are readily confined in simple cylindrical traps for relatively long periods of time, allowing for many of the experimental studies described in these proceedings. Typically, confinement is limited by inherent trap asymmetries which cause particles to move radially and be lost at the trap walls. Despite more than 20 years of study, this asymmetry-induced transport is not well understood.

In this paper, we present a brief summary of recent measurements of asymmetry-induced transport in electron plasma columns [1]. We purposely break the azimuthal symmetry by applying static voltages of strength V_a to sections of the trap wall (as shown in Figure 1), and measure the induced change in the plasma density profile. We find that the induced transport rates are well characterized by the "rigidity" $\mathcal{R} \equiv \overline{f_b}/f_E$, which is the ratio of the thermal axial bounce frequency to the azimuthal $\mathbf{E} \times \mathbf{B}$ drift frequency. Two different transport regimes are identified: "slightly-rigid" ($1 < \mathcal{R} < 10$) and "highly-rigid" ($\mathcal{R} > 20$).

In the slightly-rigid regime, the expansion rate increases linearly with the applied asymmetry strength, V_a, and decreases with rigidity as $\mathcal{R}^{-2}$. The mechanism responsible for this so-called "$V_a^1\,\mathcal{R}^{-2}$" transport appears to "turn-off" as the rigidity is increased into the range $\mathcal{R} = 10 - 20$. In the highly-rigid regime, a different transport mechanism causes transport with radically different parameter scalings.

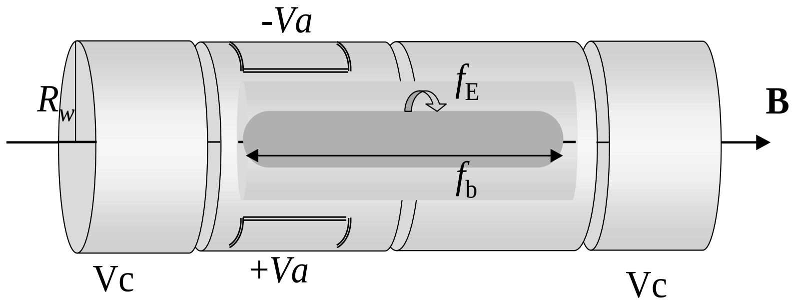

FIGURE 1. Simplified schematic of the experiment.

In this regime, the expansion rate increases with asymmetry strength as V_a^2 and is roughly independent of the plasma rigidity.

The results presented in this paper lay an empirical framework from which theory can progress, and also ties together previous experimental studies. Many previous studies measured transport due to inherent trap asymmetries in the slightly-rigid regime. These studies found that the transport also scales roughly as $\mathcal{R}^{-2}$ on many different apparatuses [2–5]. The original studies [2,3] were conducted at nearly constant density and temperature, in which case the rigidity scaling just depends on the magnetic field B and plasma length L as $\mathcal{R}^{-2} \propto L^2/B^2$. This same L^2/B^2 scaling has also been observed for particle loss measurements in experiments on test-particles in which the particle density is too low to be considered a plasma [6].

Other experiments on applied asymmetries found that the transport rate increases as the square of the asymmetry strength and only weakly depends on the magnetic field [7]. These studies were conducted on plasmas primarily in the highly-rigid regime and most of the results can be understood in terms of the scalings reported in this paper. More recent experiments have found some evidence of enhanced transport due to bounce-resonant effects [8,9], but it is not certain exactly how these experiments relate to those presented here.

ANGULAR MOMENTUM & RADIAL TRANSPORT

In a Penning-Malmberg trap (shown schematically in Figure 1), particles are prevented from leaving along the trap axis simply by applying a sufficient confining voltage V_c to end electrodes. The primary loss is in the radial direction, across the magnetic field to the walls of the trap at radius R_w. This radial transport is constrained by the conservation of angular momentum, and confinement is guaranteed in an ideal cylindrically-symmetric trap. In reality, perfect cylindrical symmetry is never achieved, and confinement is degraded by inherent trap asymmetries.

For our electron plasmas, the mechanical part of the canonical angular momentum is negligible compared to the vector potential part. Thus, the angular momentum for N_{tot} electrons with charge $-e$ can be written as

$$\mathcal{L}_\theta = \frac{eB}{2c} N_{tot} (R_w^2 - \langle r^2 \rangle), \tag{1}$$

where $\langle r^2 \rangle$ is the mean-square-radius of the plasma, and c is the speed of light. Conservation of angular momentum thus implies $d\langle r^2 \rangle / dt = 0$, and the plasma is restricted from expanding. Conversely, by breaking the symmetry, one can change $\langle r^2 \rangle$ by torquing on the plasma. Static asymmetries, such as inherent trap asymmetries, drag on the rotating plasma and cause an increase in $\langle r^2 \rangle$, *i.e.* the plasma expands. On the other hand, the so-called "rotating wall" exerts a positive torque on the plasma and causes compression by rotating faster than the plasma [10].

In this paper, the primary experimental quantity is the global expansion rate,

$$\nu_{\langle r^2 \rangle} \equiv \frac{1}{\langle r^2 \rangle} \frac{d \langle r^2 \rangle}{dt} \propto - \frac{1}{\mathcal{L}_\theta} \frac{d\mathcal{L}_\theta}{dt}. \tag{2}$$

For the experiments presented here N_{tot} is conserved. Therefore, $\nu_{\langle r^2 \rangle}$ is directly proportional to the rate of change of $\mathcal{L}_\theta$, and is unaffected by internal transport mechanisms which conserve $\mathcal{L}_\theta$, such as viscous transport [1,11].

EXPERIMENTAL MEASUREMENTS

Measurements were conducted on two different Penning-Malmberg traps known by the acronyms "CamV" and "EV". The traps are similar in construction with electrode radii of $R_w = 3.5$ cm for CamV and $R_w = 3.81$ cm for EV. Both have an excellent density diagnostic, which is crucial in accurate studies of radial transport. The primary difference is that the magnetic field on CamV has a much larger range ($B = 0.1 - 10$ kG) compared to EV ($B = 0.1 - 0.5$ kG).

The radial density profile $n(r)$ is measured by dumping the plasma out one end of the trap to a collection device. From the time evolution of the density profile we calculate local transport quantities, such as the radial flux of particles $\Gamma(r)$, as well as global quantities, such as the expansion rate $\nu_{\langle r^2 \rangle}$.

An example of the change in the density profile due to an applied asymmetry of strength $V_a = 3$ V is shown in Figure 2(a). The difference in hold times between density measurements is kept short enough that N_{tot} is conserved and the plasma density and temperature do not change substantially.

Examples of the time evolution of $\langle r^2 \rangle$ are shown in Figure 2(b) for 4 different applied voltage strengths, including $V_a = 0$. Each point is this figure is calculated from a different density profile using the formula

$$\langle r^2 \rangle \equiv \frac{2\pi\, L}{N_{tot}} \int_0^{R_w} r\, dr\, n(r)\; r^2. \tag{3}$$

At the bottom of Figure 2(b), we display the time dependence of the applied asymmetry. After the perturbation is ramped on, the plasma expands at a rate, $\nu_{\langle r^2 \rangle (V_a)}$, that depends upon the asymmetry strength V_a. With no applied voltage

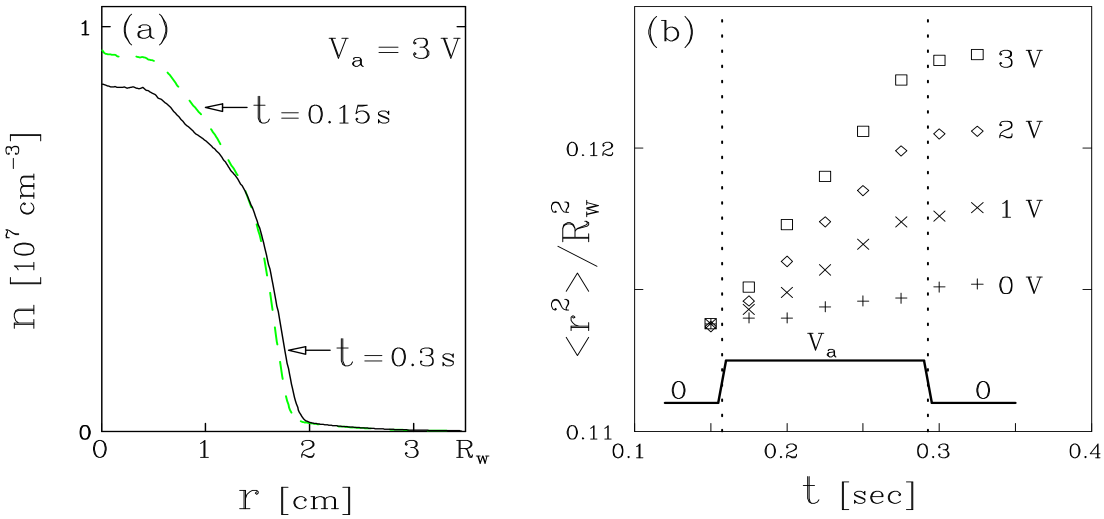

FIGURE 2. (a) Change in density due to applied asymmetry. (b) Time evolution of $\langle r^2 \rangle$ due to applied asymmetries of different strengths.

(*i.e.* $V_a = 0$), the plasma expands due to the inherent trap asymmetries. We subtract off this "background" expansion rate, $\nu_{\langle r^2 \rangle (0)}$, to obtain the net expansion rate, $\Delta\nu_{\langle r^2 \rangle (V_a)}$, due just to the applied asymmetry,

$$\Delta\nu_{\langle r^2 \rangle (V_a)} \equiv \nu_{\langle r^2 \rangle (V_a)} - \nu_{\langle r^2 \rangle (0)} \tag{4}$$

We find that this expansion rate is well characterized by the plasma rigidity $\mathcal{R}$. In terms of the axial magnetic field B, the average plasma density n, length L, and temperature T, we approximate the rigidity for each plasma as

$$\mathcal{R} \equiv \overline{f_b}/f_E \approx 14.6 \left[\frac{B}{1\,\mathrm{kG}}\right] \left[\frac{T}{1\,\mathrm{eV}}\right]^{1/2} \left[\frac{n}{10^7\,\mathrm{cm}^{-3}}\right]^{-1} \left[\frac{L}{10\,\mathrm{cm}}\right]^{-1}. \tag{5}$$

EXPERIMENTAL RESULTS

The voltage dependencies of the expansion rate are shown in Figure 3 for the two different transport regimes. For the slightly-rigid plasma ($\mathcal{R} = 2.5$) shown in Figure 3(a), the expansion rate increases linearly with applied voltage (*i.e.* $\Delta\nu_{\langle r^2 \rangle} \propto V_a^1$) over two orders of magnitude in V_a. This scaling is in direct contradiction to current theories of transport due to bounce-resonant particles, which predict either a V_a^2 or $V_a^{1/2}$ dependence [9,12]. The arrows at the bottom of the figure indicate the effective strength of inherent trap asymmetries for EV and CamV as well as published estimates for traps at Occidental College [6] and the University of California at Berkeley [7]. Figure 3(a) thus shows that the V_a^1 scaling occurs for

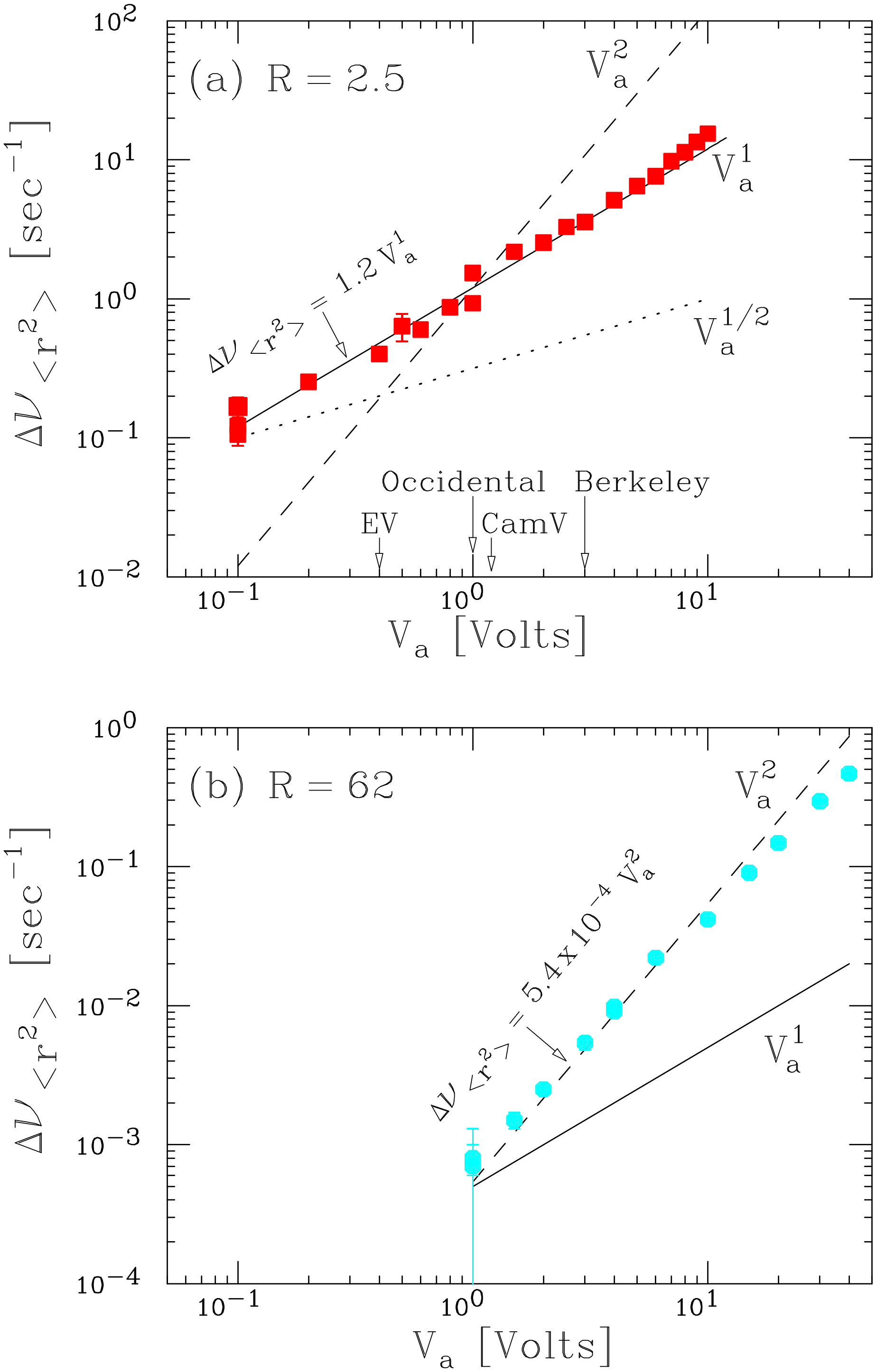

FIGURE 3. Net expansion rate vs asymmetry strength for (a) a slightly-rigid plasma (b) a highly-rigid plasma.

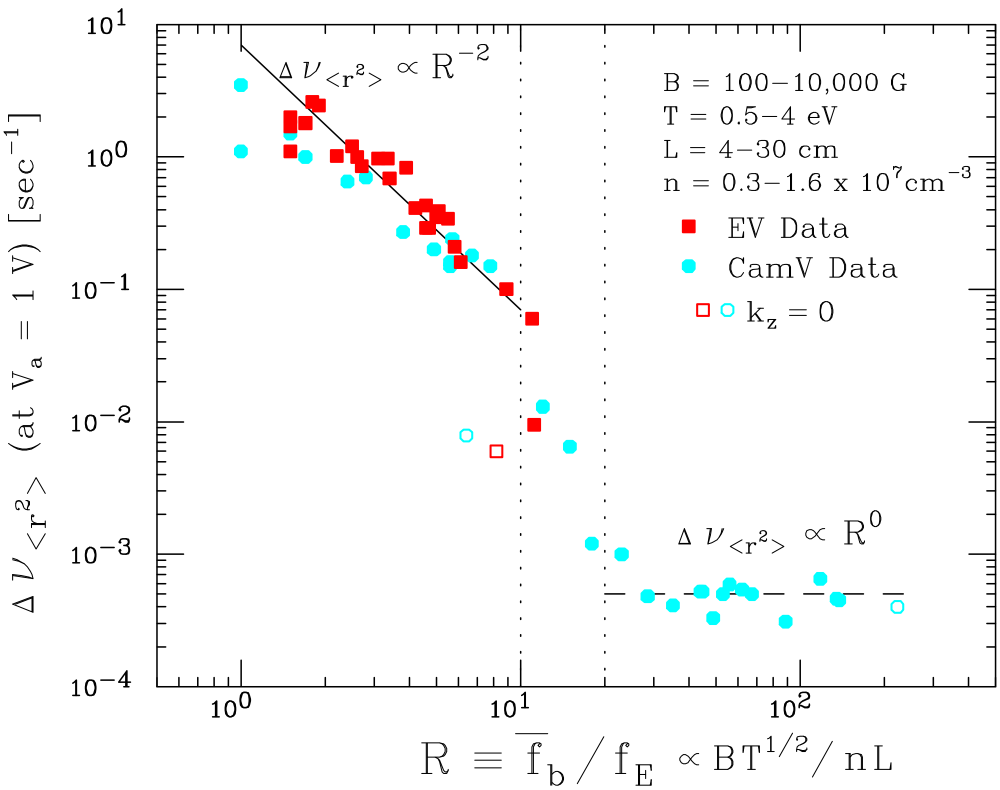

FIGURE 4. Net expansion rate at $V_a = 1$ V vs the plasma rigidity.

asymmetry strengths on the order of (and even less than) the inherent asymmetries in many typical Penning-Malmberg traps.

The voltage scaling is different for a highly-rigid plasma. The expansion rate for these plasmas increases as $\Delta\nu_{\langle r^2\rangle} \propto V_a^2$ even for relatively large asymmetry strengths, as shown in Figure 3(b) for a plasma with $\mathcal{R} = 62$.

In addition to the voltage dependence, we have measured asymmetry transport for a range of density, length, temperature, and magnetic field. The scaling of the expansion rate with these plasma parameters is condensed to a dependence on the rigidity, as shown in Figure 4. The points in this figure are obtained from measurements of $\Delta\nu_{\langle r^2\rangle}$ vs V_a for each set of initial conditions. The voltage dependence is scaled out by fitting each data set to a power-law; for a slightly-rigid plasma we fit to V_a^1, and for a highly-rigid plasma we fit to V_a^2. We evaluate the data fits at $V_a = 1$ V, and plot the results in Figure 4 as a function of the rigidity of the plasma. For example, fits to the data sets shown in Figure 3 yield values of 1.2 for $\mathcal{R} = 2.5$ and 5.4×10^{-4} for $\mathcal{R} = 62$.

In Figure 4, the expansion rate for slightly-rigid plasmas is shown to decrease with plasma rigidity approximately as $\Delta\nu_{\langle r^2\rangle} \propto \mathcal{R}^{-2} \propto \frac{n^2 L^2}{T B^2}$. In this regime, the expansion rate is well approximated with the simple formula [1]

$$\Delta\nu_{\langle r^2\rangle} = 7\,\mathrm{sec}^{-1} \quad \left[\frac{V_a}{1\mathrm{V}}\right] \mathcal{R}^{-2} \quad (\mathrm{slightly-rigid}), \tag{6}$$

where $\mathcal{R}$ is approximated by Equation 5.

As the plasma rigidity is increased, not only does the voltage dependence change (as shown in Figure 3), but the dependence on plasma parameters changes also. The $V_a^1\,\mathcal{R}^{-2}$ mechanism appears to "turn-off" as the rigidity is increased into the range $\mathcal{R} \gtrsim 10$. This "turn-off" is not understood, but may be an indication of the bounce motion becoming a good adiabatic invariant at high rigidity.

For a highly-rigid plasma, the transport is independent of rigidity, as shown by the circular points on the bottom right of Figure 4. For this data, the rigidity was primarily varied by changing the magnetic field; therefore, the independence in rigidity is more accurately an independence in magnetic field (*i.e.* $\Delta\nu_{\langle r^2\rangle} \propto B^0$).

Cross-field transport that is independent of the magnetic field strength has been observed in other experiments which measured the damping of the $m = 1$ diocotron mode [13]. These measurements were found to agree very closely with so-called "Rotational-Pumping" theory [14]. A similar mechanism may be causing asymmetry-induced transport in the highly-rigid regime; however, calculations of Rotational-Pumping theory appropriate to the perturbations applied here have not yet been made.

In addition to the observed independence with magnetic field, the expansion rate in the highly-rigid regime has been found to be roughly independent of length and inversely proportional to both density and temperature. For this (somewhat preliminary) data, the transport rate is approximated with the formula [1]

$$\Delta\nu_{\langle r^2\rangle} \approx 5\times10^{-4}\,\mathrm{sec}^{-1} \quad \left[\frac{V_a}{1\mathrm{V}}\right]^2 \left[\frac{T}{1\mathrm{eV}}\right]^{-1} \left[\frac{n}{10^7\mathrm{cm}^{-3}}\right]^{-1} \quad (\mathrm{highly-rigid}). \tag{7}$$

DEPENDENCE ON AXIAL AND AZIMUTHAL VARIATIONS

In the remainder of this paper, we present results concerning the dependence of the measured transport on the axial and azimuthal variations of the applied asymmetry. We begin by considering the decomposition of an asymmetry applied at $r = R_w$ into Fourier vacuum fields of relative amplitude A_{m,k_z}:

$$\Phi_a(r,\theta,z) = V_a \sum_{m,k_z} A_{m,k_z} \left(\frac{r}{R_w}\right)^m e^{i\,m\,\theta}\, e^{i\,\pi\,k_z\,z/L}, \tag{8}$$

where k_z describes the axial dependence and m describes the azimuthal dependence.

For all the data shown in Figures 3 and 4, a positive voltage $+V_a$ was applied to a wall patch on one side of the plasma and a negative voltage $-V_a$ was applied to a patch on the opposite side (as shown in Figure 1). We label this an $m = 1$

asymmetry, in terms of its dominant azimuthal mode number. The wall patches are of fixed length of about 4 cm. In general, the plasma extends beyond the patches (as shown in Figure 1), which means the plasma "sees" an asymmetry with an axial variation. Properly considered, the asymmetry has many different k_z Fourier modes, but we simply refer to this case as a $k_z \neq 0$ asymmetry. The data in Figure 3 and all the solid points in Figure 4 are for $k_z \neq 0$ asymmetries.

We find that the $V_a^1 \mathcal{R}^{-2}$ transport mechanism is not active if we apply the asymmetry over the entire axial extent of a short ($L \approx 4$ cm) plasma. In this case, $k_z = 0$ and Equation 6 is not valid. Instead, the expansion rate follows a $\Delta\nu_{\langle r^2 \rangle} \propto V_a^2$ dependence regardless of whether the plasma is slightly- or highly-rigid. Also, the magnitude of the expansion rate in the slightly-rigid regime is about a factor of 10 less for a $k_z = 0$ compared to a $k_z \neq 0$ asymmetry of $V_a = 1$ V. This latter result is shown in Figure 4, where the open points are for $k_z = 0$. Therefore, it appears that the plasma must be slightly-rigid *and* the asymmetry must have some axial variation for the $V_a^1 \mathcal{R}^{-2}$ mechanism to be active.

In addition to measurements for $m = 1$ asymmetries, we have also measured transport due to nominal $m = 2$ and $m = 4$ asymmetries. We find that the *global* expansion rate $\Delta\nu_{\langle r^2 \rangle}$ does not depend strongly on the azimuthal variation in the asymmetry [1]; however, the radial dependence of *local* transport measurements are qualitatively different.

In Figure 5, we shown the effects on the density profile $n(r)$ of a slightly-rigid plasma ($\mathcal{R} = 2.5$) due to the inherent trap asymmetries, and due to $m = 1$, $m = 2$, and $m = 4$ applied asymmetries. Here the asymmetry is $k_z \neq 0$ applied near the axial middle of a long ($L = 30$ cm) plasma. In all four cases, the plasma has the same initial density profile (dashed curves), but the different asymmetries cause the plasma to evolve to different final density profiles (solid curves). The time between profiles is $\Delta t = 0.033$ sec, and we plot $\Delta n(r)/\Delta t$ in the top portion of each figure.

The inherent asymmetries cause a relatively small amount of transport only at the radial edge of the plasma. In contrast, an $m = 1$ asymmetry causes transport across the the entire plasma. The $m = 2$ and $m = 4$ asymmetries cause a similar amount of transport near the edge of the plasma, but the density near the center of the plasma is not strongly affected. The contrast is particularly striking between the $m = 1$ and $m = 4$ asymmetries.

From the measured change in density, we calculate the local radial flux of particles for the different m number asymmetries as $\Gamma(r) \equiv -\frac{1}{r}\int_0^r dr'\, r'\, \frac{\Delta n(r')}{\Delta t}$. The fluxes are shown in Figure 6 to vary as $\Gamma(r) \propto r^m$ over the interior of the plasma. The dashed curve in the figure is the initial density profile, which is plotted to show that the flux goes to zero at the edge of the plasma (as expected).

In the slightly-rigid regime, not only does the global transport depend linearly on the applied voltage (*i.e.* $\Delta\nu_{\langle r^2 \rangle} \propto V_a$), but the local flux appears to vary linearly with the local vacuum field of the asymmetry (*i.e.* $\Gamma(r) \propto V_a\,(r/R_w)^m$). This is somewhat surprising, because one might have expected the asymmetry to be "shielded" from the center of the plasma, and the transport to occur just at the

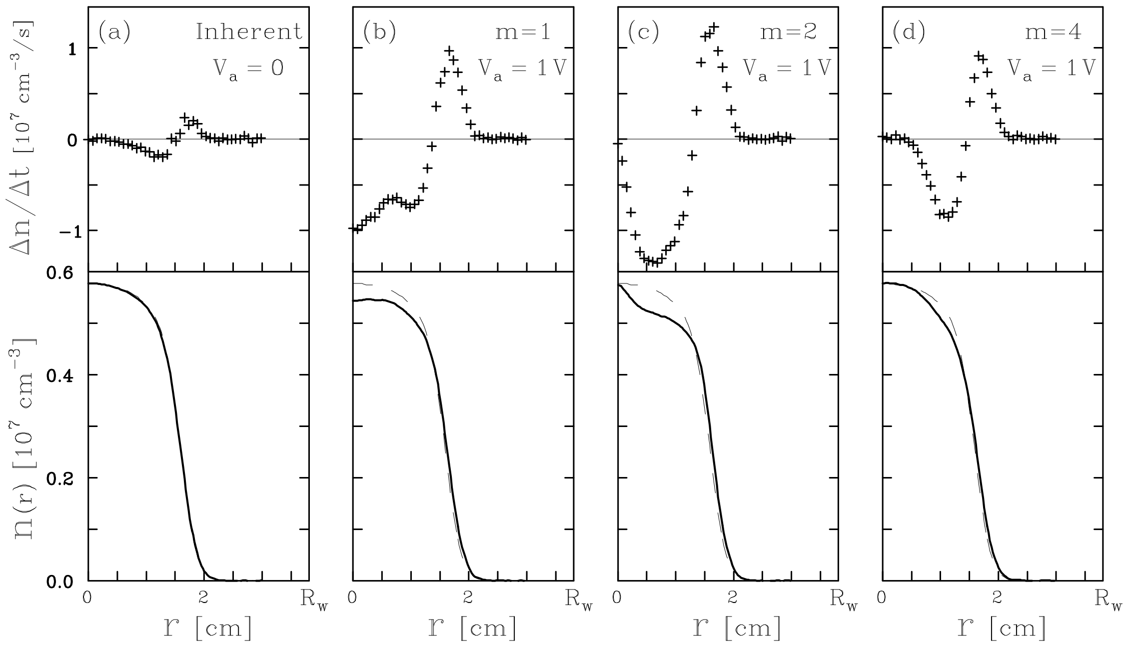

FIGURE 5. Change in density profile for a slightly-rigid plasma ($\mathcal{R} = 2.5$) over a short duration ($\Delta t = 0.033s$) due to (a) inherent trap asymmetries (b) applied $m = 1$ asymmetry (c) applied $m = 2$ asymmetry (d) applied $m = 4$ asymmetry.

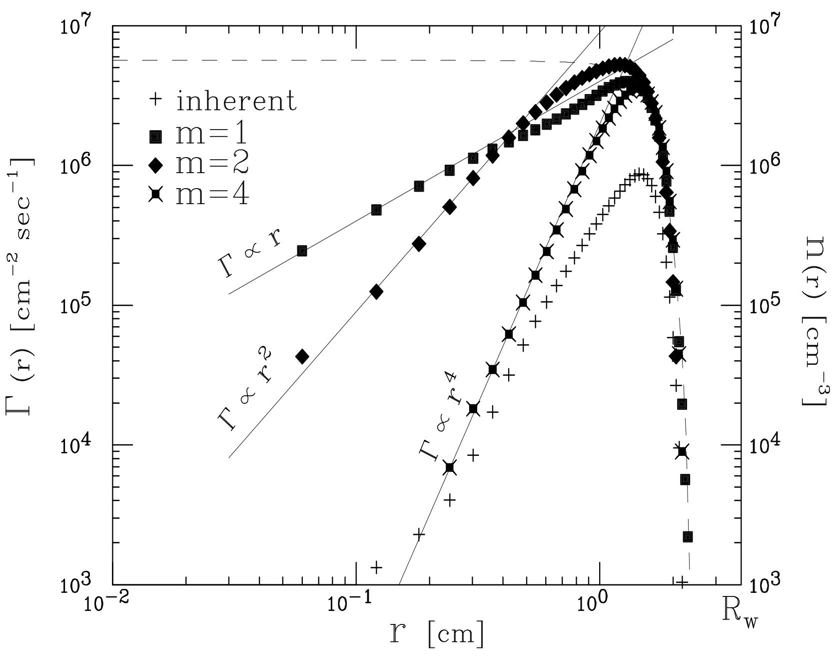

FIGURE 6. Radial dependence of the particle flux for different m-number asymmetries.

radial edge of the plasma. However, as was shown in Figure 5(b), transport can occur over the entire plasma with no apparent effects due to shielding.

SUMMARY

We measure transport caused by static azimuthally asymmetric voltages applied to wall patches on cylindrical traps. We identify two different transport regimes, slightly-rigid and highly-rigid, in which two different mechanisms dominate the transport. The parameter dependence in these two regimes are summarized in the table below. In terms of the applied asymmetry strength and rigidity, plasmas expand at a rate that scales as $\Delta\nu_{\langle r^2\rangle} \propto V_a\,\mathcal{R}^{-2}$ in the slightly-rigid regime. This mechanism is not active when the rigidity is too high, nor when the asymmetry is applied over the entire plasma. In the highly-rigid regime, plasma expansion scales as $\Delta\nu_{\langle r^2\rangle} \propto V_a^2\,\mathcal{R}^0$, which is independent of the magnetic field strength.

Regime	Voltage Dependence	Parameter Dependence
Slightly Rigid ($1 < \mathcal{R} < 10$)	V_a^1	$\mathcal{R}^{-2} \propto \frac{n^2\,L^2}{T\,B^2}$
Highly Rigid ($\mathcal{R} > 20$)	V_a^2	$\frac{B^0\,L^0}{n\,T}$ (preliminary)

This work supported by ONR #N00014-96-1-0239 and NSF #PHY-9876999.

REFERENCES

1. Kriesel, J. M., *PhD thesis, University of California at San Diego*, (1999).
2. Driscoll, C. F. and Malmberg, J. H., *Phys. Rev. Lett.* **50**, 167 (1983).
3. Driscoll, C. F., Fine, K. S., and Malmberg, J. H., *Phys. Fluids* **29**, 2015 (1986).
4. Cluggish, B. P., *PhD thesis, University of California at San Diego*, (1995).
5. Cass, A. C., *et. al.*, *Bull. Am. Phys. Soc.* **39**, 1737 (1994).
6. Eggleston, D. L., *Phys. Plasmas* **4**, 1196 (1997).
7. Notte, J., Fajans, J., *Phys. Plasmas* **1**, 1123 (1994).
8. Gilson, E., Fajans, J., *see article in these proceedings.*
9. Eggleston, D. L., *see article in these proceedings.*
10. Anderegg, F., Hollmann, E. M., and Driscoll, C. F., *see article in these proceedings.*
11. Kriesel, J. M. and Driscoll, C. F., *see separate article in these proceedings.*
12. Eggleston, D. L. and O'Neil, T. M., *Phys. Plasmas* **6**, 2699 (1999).
13. Cluggish, B. P. and Driscoll, C. F., *Phys. Rev. Lett.* **74**, 4213 (1995).
14. Crooks, S. and O'Neil, T. M., *Phys. Plasmas* **2**, 355 (1995).