Experimental Test of the Quasilinear Theory of the Interaction between a Weak Warm Electron Beam and a Spectrum of Waves

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Recent theoretical work has suggested that the quasilinear description of the weak-warm-beam–plasma instability is incomplete. It has been argued that beam-mediated mode-coupling effects neglected in standard quasilinear theory play an important role, and that their average effect is a zeroth-order increase in the growth rate. We have experimentally observed strong mode-coupling effects when a weak warm beam interacts with waves on a slow-wave structure. However, when we average the effects of the mode coupling we do not observe the predicted zeroth-order increase in the growth rate.

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One of the simplest examples of the development of turbulence in plasmas is the weak-warm-beam–plasma instability (also known as the Gentle bump on tail instability). In this interaction a low-density, warm electron beam is injected into a plasma causing a spectrum of modes to become unstable and to grow at the expense of the beam kinetic energy. In the traditional quasilinear description[1] the mode growth rate is proportional to the slope of the time-averaged velocity distribution function evaluated at the phase velocity of the mode. As the waves grow, energy is extracted from the beam in such a way as to reduce the slope of the distribution function. Saturation occurs when the slope is reduced to zero, thus forming a plateau in the velocity distribution function.

Fifteen years ago, a detailed experimental test of quasilinear theory was performed by Roberson and Gentle.[2] However, they were unable to directly check a crucial approximation made in quasilinear theory—the neglect of mode coupling mediated by the beam. Since the beam dynamics are highly nonlinear, one might expect that there would be contributions to the mode electric field as a result of nonlinear products of the other modes. These contributions are neglected in the quasilinear description. The neglect of mode-coupling effects has been strongly criticized[3,4] in the past. Furthermore, it has been suggested[5-7] that when the mode-coupling interactions are included in the theory, their average effect is to cause a zeroth-order increase in the growth rates of the modes.

An important feature of the weak-warm-beam–plasma instability is that if the beam is of sufficiently low density, then the background plasma behaves as a linear dielectric and acts only to support the waves. We exploit this feature in our experiment by replacing the plasma with a slow wave structure. This replacement preserves the basic physics of the instability and ensures the mathematical correspondence between the warm-beam slow-wave structure interaction and the warm-beam–plasma interaction. Thus, we inject a warm electron beam through a helical slow-wave structure.[8] With this device we have observed strong mode-coupling effects which are neglected in standard quasilinear theory. However, when we average over these mode-coupling effects, we do not observe the predicted zeroth-order increase in the growth rate.

The interaction between the beam electrons and the growing spectrum of waves is characterized by the ratio of the particle autocorrelation length to the spatial growth length, $\eta_p = k_i \omega / k^2 \Delta v_p$, and by the ratio of the field autocorrelation length to the spatial growing length, $\eta_s = k_i / \Delta(k - \omega/v_\phi) = k_i/(l/v_g - l/v_\phi)\Delta\omega$. Here ω, k, and k_i are the angular frequency, wave number, and spatial growth rate of the mode. Δv_p represents the width of the distribution function; in our experiment we define $\Delta v_p = v_{75} - v_{25}$, where v_{75} and v_{25} are the velocities at which the unperturbed beam parallel-energy distribution function has decreased down its positive slope to 75% and 25% of its maximum value. v_g and v_ϕ are typical group and phase velocities of waves within the bandwidth, $\Delta\omega$, of the spectrum. In the experiment, $0.038 < \eta_p < 0.098$ and $0.17 < \eta_s < 0.36$.

The experimental apparatus[9] consists of an electron beam which is confined by a strong magnetic field ($B_z = 440$ G) and directed along the axis of a helical slow-wave structure. The electron beam, which is pulsed at a 60-Hz repetition rate with a typical pulse width of 400 μsec, enters the slow-wave structure with controllable axial-velocity spread. The spread is produced[10] by passing a cold beam through three parallel, closed-spaced, wire mesh grids biased at potentials 0, V_s, and 0. Outside of the glass vacuum tube are axially movable electrostatic probes which are used to transmit and receive radio frequency waves. After passing though the helix, the beam enters a retarding-field analyzer. With this device we have observed the growth and saturation of launched broadband noise. In Fig. 1(a) we show a plot of the logarithm of the total received power as a function of distance, z, down the tube. The wave power is seen to grow exponentially nearly 15 dB and then saturate. In Fig. 1(b) we show the corresponding evolution

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of the time-averaged parallel-energy distribution function of the beam. As the noise grows and saturates, the beam distribution is seen to evolve into a plateau.

There is an experimental advantage to using a slow-wave structure rather than a plasma to support the wave propagation. Unlike a plasma, the helix does not introduce any appreciable noise. In a beam-plasma system low-frequency ion noise causes phase jitter in the unstable spectrum which makes mode-coupling measurements very difficult. By replacing the plasma with a slow wave structure we eliminate these undesirable effects. In addition, in the beam–slow-wave-structure system the background noise level is so low that it is possible to measure the growth rate of a single wave launched far below saturation. This single-wave growth rate is very useful, and since it is the only wave in the system, no mode coupling is possible; it is thus an experimental definition of the Landau growth rate.

For the case shown in Fig. 1, we launched a spectrum of nonrepetitive noise (derived from the input noise of an amplifier). A more interesting kind of spectrum to launch is repetitive "noise." By repetitive noise we mean an arbitrary but well defined time-varying signal that lasts for some time, T, and that then repeats. This time-varying signal could, for example, be a sample of duration T taken from truly random noise such as band-limited Johnson noise. T is generally chosen to be a time longer than the longest physically relevant time in an experiment. In our case, this is the beam transit time ($\sim 0.5\ \mu$sec). We manufacture this repetitive noise using a computer-controlled, high-speed arbitrary wave-form generator which was built for this purpose and which has been described in detail elsewhere.[11] This device allows us to prescribe the complex Fourier coefficient of each mode of the launched spectrum. The use of repetitive noise enables us to study this turbulent system with all relevant initial conditions under our control and with all genuine randomness eliminated from the dynamical variables that describe the system.

With the aid of the arbitrary wave-form generator we have observed strong mode-coupling effects. Figure 2(a) shows a typical launched spectrum of repetitive noise. The amplitudes of the modes (which are discrete[12] since the signal is repetitive with a period of 2.56 μsec) have been chosen to be a smooth function of frequency, but their phases have been chosen by a random number generator in the computer. Figure 2(a) shows the spectrum measured upstream, near the transmitter. Figure 2(b)

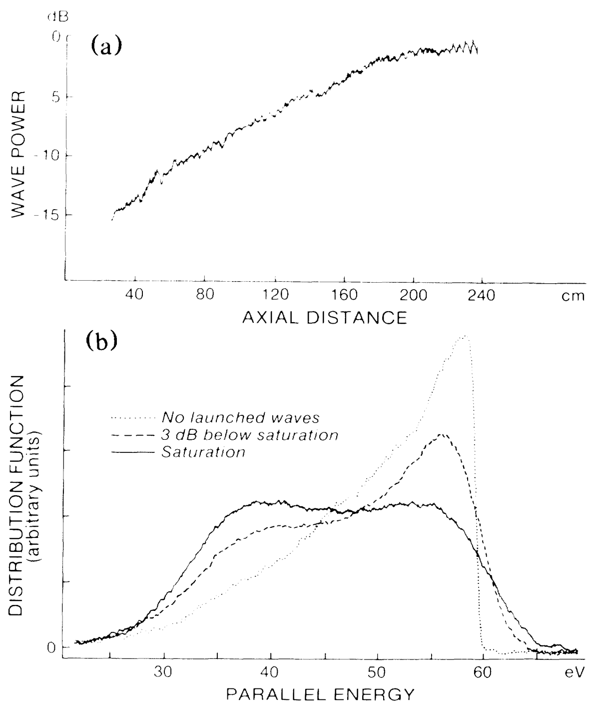

FIG. 1. (a) Total wave power vs axial distance. Beam voltage is $V_b = 60.0$ V, beam current is $I_b = 110\ \mu$A, and spreader bias voltage is $V_s = 2.0$ kV. $\eta_p = 0.065$ and $\eta_s = 0.17$. (b) Time-averaged beam parallel-energy distribution function at various positions vs parallel energy. Same parameters as (a). The launch level is held constant.

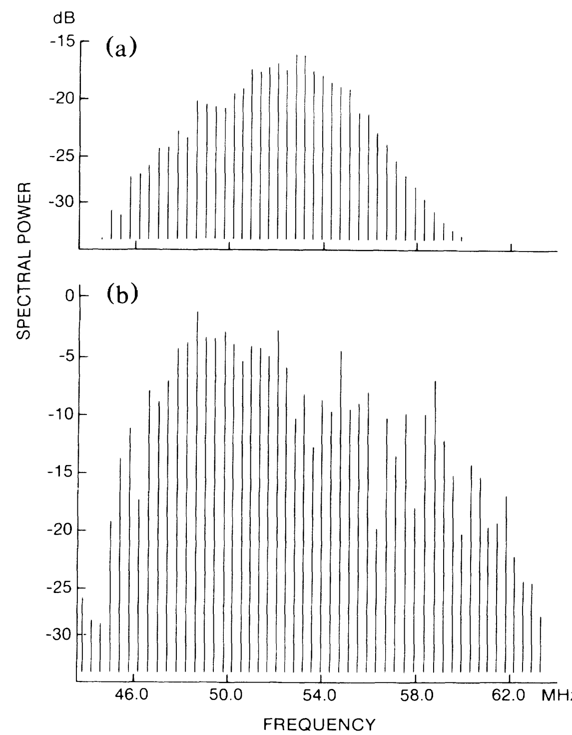

FIG. 2. (a) Spectrum launched by the arbitrary wave-form generator. The spectral lines shown here [and in Figs. 3(b) and 4(a)] are meant to show only the measured power at each mode frequency. The finite linewidth at each frequency has been suppressed for clarity. (b) Spectrum downstream. $V_b = 60.0$ V, $I_b = 140\ \mu$A, $V_s = 2.5$ kV, $\eta_p = 0.098$, and $\eta_s = 0.36$.

shows the spectrum downstream. The modes have grown because of their interaction with the beam and most of them have saturated. However, their amplitudes are no longer the smooth function of frequency predicted by standard quasilinear theory. This nonsmooth behavior has also been seen in computer simulations.[6,13] The nature of this behavior was investigated further by launching a spectrum which was similar to that of Fig. 2(a) but with one of the modes in the middle of the spectrum having a level 20 dB below the neighboring modes. This spectrum is shown in Fig. 3(a). The solid line in Fig. 3(b) shows how this mode evolves as a function of axial distance down the tube. Up to $z=100$ cm the mode grows at the single-wave growth rate. At $z=100$ cm the wave exhibits a change in growth rate and continues to grow at an enhanced growth rate until it saturates. This growth enhancement is due to mode coupling into this frequency caused by the surrounding larger modes. The surrounding large modes produce an induced electric field which is here adding in phase to the electric field of the launched mode. The dotted curve shows the evolution when we change the phase of the small launched mode by 180° and leave the phases and amplitudes of all the other modes the same. Again the mode grows at the single-wave growth rate until about 90 cm where it experiences a dramatic dip in power as the induced electric field caused by mode coupling interferes destructively with the electric field of the launched mode. The fast oscillations seen in this plot are due to a beat between the forward wave and a small component of backward wave originating from reflections from the end of the helix and from irregularities in the windings. Since the backward wave is far out of synchronism with the beam, its interaction with the beam is negligible. In addition, there are also very weak slow oscillations visible in the plot, especially in the region between 40 and 100 cm. These are due to a beat between the forward wave on the helix and a passive mode on the beam. We regard these small amplitude oscillations as incidental to the main effect shown in Fig. 3, namely, that the mode coupling can cause quite dramatic effects on the evolution of the modes.

When we average over these mode-coupling effects, we do not see the predicted zeroth-order increase in the growth rate. To demonstrate this we first launch a smooth spectrum of modes using the arbitrary wave-form generator [similar to the spectrum of Fig. 2(a)]. The dashed line in Fig. 4 shows the evolution of one of the modes. Strong mode-coupling effects are seen to occur beginning at $z=80$ cm. Next we replace the wave-form generator with a nonrepetitive noise source.

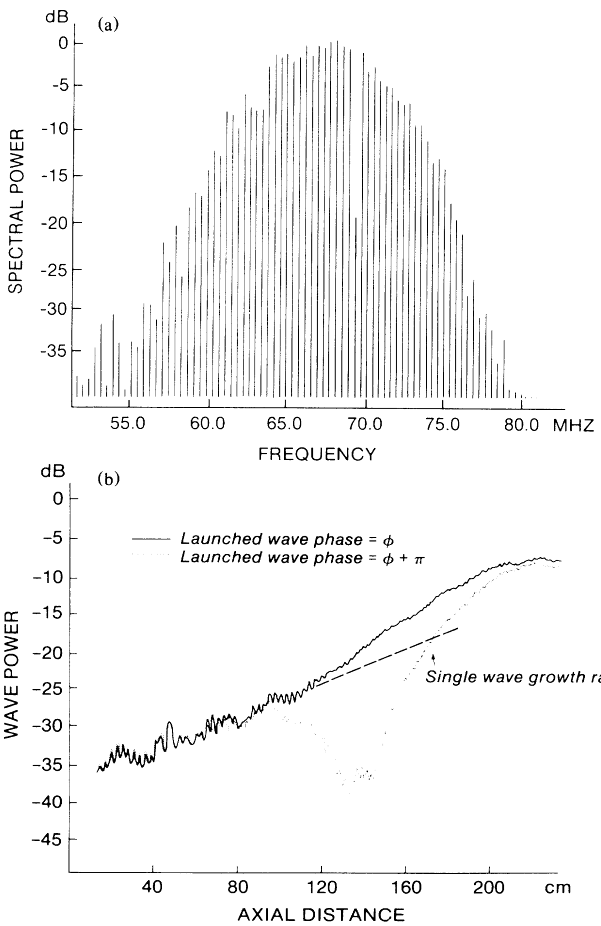

FIG. 3. (a) Spectrum used to further investigate the mode coupling. (b) Wave power vs axial distance of a mode with phase equal to ϕ (solid curve) and of the mode with phase equal to $\phi+\pi$ (dotted curve). $V_b=60.0$, $I_b=110$ μA, $V_s=2.0$ kV, frequency, $f=69.12$ MHz. $\eta_p=0.038$ and $\eta_s=0.20$.

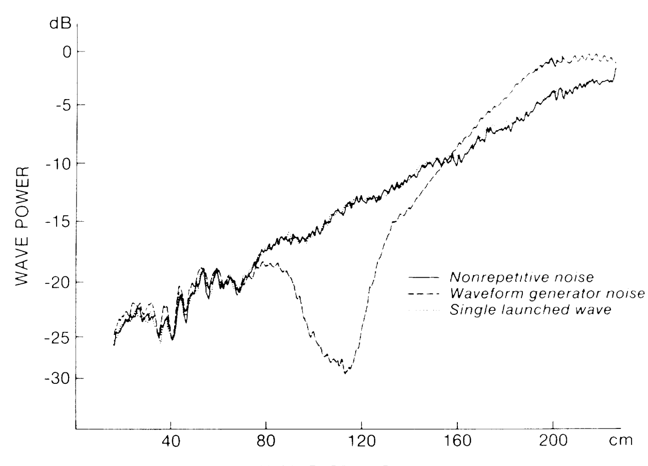

FIG. 4. Wave power vs axial distance for various launched signals: Repetitive noise (dashed curve), nonrepetitive noise (solid curve), and a single wave (dotted curve). $V_b=60.0$, $I_b=140$ μA, $V_s=2.5$ kV, and $f=62.34$ MHz. $\eta_p=0.055$ and $\eta_s=0.19$.

The nonrepetitive noise is similar to the wave-form generator noise in both shape and magnitude. We receive with a narrow-band receiver (bandwidth = 100 kHz) which squares the received electric field and then time averages it (averaging time > 100 μsec). Thus, during each beam pulse, a frequency average over the receiver bandwidth is taken. These frequency-averaged values are then ensemble averaged over many beam pulses. The solid curve shows the evolution of the logarithm of this average. The dotted curve which is superimposed upon it is the evolution of a single launched wave (our experimental definition of the Landau growth rate). To good accuracy (better than 10%) no deviation from the Landau growth rate is seen.

To summarize, we have observed the saturation of the modes and the development of the plateau in the beam distribution function when a weak warm beam interacts with waves on a slow-wave structure. We have observed strong mode-coupling effects which are neglected in standard quasilinear theory. However, when we average over these mode-coupling effects, we do not observe the predicted zeroth-order increase in the growth rate. The observation of strong mode coupling indicates that one cannot *a priori* neglect mode-coupling terms in the theory. The absence of any zeroth-order change in the growth rate when we average over the mode-coupling effects might be explained by (1) some unknown but crucial difference between temporally and spatially growing systems, (2) some subtle difference between our finite-radius, warm-beam–slow-wave-structure system and the infinite, warm-beam–plasma system, or (3) the incompleteness of the current theoretical ideas[5–7]; in fact, the experimental results suggest the action of some statistical or dynamical conservation law whose origin has not yet been theoretically identified.

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