

Nonneutral Plasmas Have Exceptional Confinement Properties

Recent experiments have achieved impressively long confinement times for nonneutral plasmas. The experiments are discussed briefly and the theoretical reasons for expecting long confinement times are presented.

Recent experiments have involved the confinement by static electric and magnetic fields of a column of unneutralized electrons.¹ The column may be referred to as a plasma (i.e., a pure electron plasma) by the criterion that the Debye length is small compared to the radius of the column. For a magnetic field in the range of 1 kG the confinement time is in the range of 1 min (see Figure 2); and there is reason to believe that a magnetic field of 100 kG will yield a confinement time in the range of an hour, or perhaps even a day. Confinement times in excess of a day have already been achieved for experiments involving small numbers of electrons.² However, these experiments were designed to study questions in atomic physics, and there was no motivation to push the density into the plasma regime.

The confinement geometry for the experiments of Ref. 1 is shown schematically in Figure 1. The geometry is cylindrical and the entire system is immersed in a uniform axial magnetic field, B_0 . The system is pulsed repetitively in the following sequence. (1) Initially, conducting cylinder A is grounded and C is biased sufficiently negative to reflect all electrons emitted from the negative biased source. During this filling period, a column of electrons occupies cylinders A and B. (2) The potential of cylinder A is then gated negative to cut off the flow of incoming electrons and to trap those electrons in B. The electrons in B are confined radially by the magnetic field and axially by electrostatic fields between B and the negatively biased end cylinders, A and C. (3) In the course of time, some electrons wander across the magnetic field and hit the cylindrical wall, where they are absorbed. (4) A time t after trapping, cylinder C is gated to ground potential, dumping the remaining electrons out along the magnetic field lines.

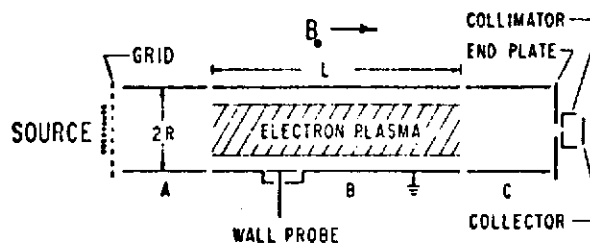


FIGURE 1. Schematic diagram of the confinement geometry for the experiments of Ref. 1.

A radially moveable collector assembly collects the electrons at its radial position. From the charge collected, the area of the collector and the length of the column, the density $n(r, t)$ is inferred.

For a sample run, the initial density is $n = 1.4 \times 10^7 \text{ cm}^{-3}$, yielding a plasma frequency of $\omega_p = (4\pi n e^2 / m)^{1/2} \approx 2.1 \times 10^8 \text{ rad/s}$. The magnetic field is 675 G, yielding a cyclotron frequency of $\Omega = eB_0 / mc \approx 1.2 \times 10^{10} \text{ rad/s}$. Thus, the density is well below the Brillouin limit⁴ (i.e., $\omega_p / \Omega \approx 2 \times 10^{-2} \ll 1$). The temperature is measured by passing the dumped electrons through a velocity analyzer. Since the component of velocity parallel to the magnetic field is changed significantly during disassembly of the plasma, the analyzer is designed to measure the perpendicular velocity. The initial temperature is in the range of 1 eV. Combining this with the density $n \approx 1.4 \times 10^7 \text{ cm}^{-3}$ yields a Debye length of $\lambda_D \approx 0.2 \text{ cm}$, which is small compared to the initial radius of plasma, $r \approx 1.4 \text{ cm}$.

Figure 2 shows the number of electrons per unit length remaining after time t — i.e., $N(t) = \int_0^R 2\pi r dr n(r, t)$, where R is the radius of the cylinder. The electrons begin to hit the wall after about 30 s. One can define a confinement time τ as a function of B_0 and P , where P is the pressure of the ambient neutral gas. In the high-pressure regime, the electron-loss mechanism is cross-magnetic-field transport due to electron-neutral collisions, and the scaling of the confinement time with P and B_0 is classical (i.e., $\tau \propto B^2 / P$).^{5,6} The longest confinement times (e.g., Figure 2) are obtained for low neutral pressure, where the loss mechanism is not understood clearly. In this regime, the empirically determined scaling of confinement time with magnetic field is approximately $\tau \propto B_0^{-1.5}$. Extrapolating from 1 to 100 kG raises τ from 10^2 to 10^5 s. Naturally, a lot can go wrong in extrapolating the field over two orders of magnitude, but the predicted confinement time (i.e., a day) is certainly striking.

Next, let us look at the confinement problem from a theoretical point of view. The axial confinement can be guaranteed simply by making the bias on the end cylinders sufficiently negative. To understand the radial confinement, it is useful to introduce the total canonical angular momentum for the electrons

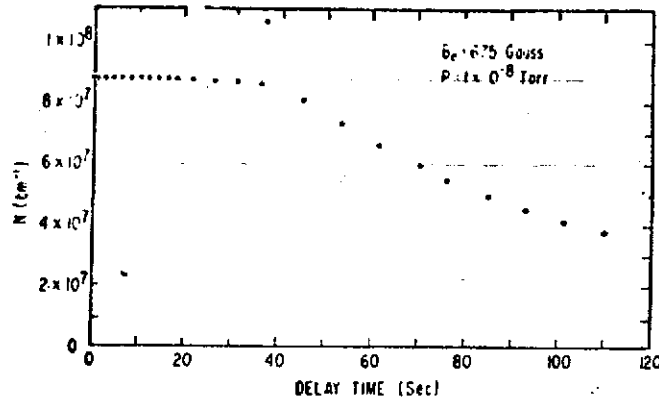


FIGURE 2. Number of electrons per unit length — i.e., $N(t) = \int_0^R 2\pi r dr n(r, t)$ — versus the time since trapping.

$$P_\theta = \sum_j [mv_{\theta j} r_j - \frac{e}{c} A_\theta(r_j) r_j]. \quad (1)$$

Here, $v_{\theta j}$ is the θ component of the velocity for the j^{th} electron and r_j is the radial position of the j^{th} electron. For a uniform magnetic field, the θ component of the vector potential is given by $A_\theta(r) = B_0 r/2$. The diamagnetic field is negligible for the low electron densities and velocities considered here (i.e., $\omega_p \ll \Omega$ and $v_\theta \ll c$). The ratio of the mechanical part of the angular momentum to the vector-potential part of the angular momentum is given by $(mv_{\theta j} r_j)/(eB_0 r_j^2/2c) = 2v_{\theta j}/\Omega r_j$. The velocity v_θ is composed of two parts: a drift component $cE/B \sim (\omega_p^2/\Omega)r_j$ and a thermal component $\sqrt{kT/m}$. For the experimental conditions mentioned above, one can check that $v_{\theta j}/\Omega r_j \ll 1$; so Eq. (1) can be rewritten as

$$P_\theta \approx - \sum_j (eB_0/2c) r_j^2. \quad (2)$$

More formally, conservation of energy may be introduced to bound $\sum mv_{\theta j} r_j$ and show that Eq. (2) is a good approximation whenever it is a good approximation initially and $\omega_p \ll \Omega$.⁷ Since there is only a single charge species, e may be factored out from under the sum along with the other constants. To the extent that P_θ is conserved (recall that the geometry has cylindrical symmetry), there is a constraint on the allowed radial positions for the electrons, $\sum r_j^2 = \text{const}$. If some electrons move out, others must move in. As an example, suppose that the electrons are injected at a radius of 1 cm, that the cylindrical wall is at a radius of 10 cm and that 1% of the electrons go to the wall. The other 99% of the electrons must go to the axis in order to satisfy $\sum r_j^2 = \text{const}$. Even if the plasma is violently unstable, only a small fraction of the electrons can be lost provided P_θ is conserved.

Examples of effects that can change P_θ are electron-neutral collisions, finite wall resistance and field errors that are not cylindrically symmetrical. These effects can apply a net torque on the electrons and produce a continuing radial transport to the wall. As mentioned above, the transport observed in the experiments of Ref. 1 has been identified as being due to electron-neutral collisions, in the regime of high neutral pressure. In the regime of low neutral pressure, there are some indications that the transport is due to field errors. Electromagnetic radiation can remove only the small amount of angular momentum associated with the thermal motion of the electrons, provided the radius of the cylinder is chosen to satisfy the inequality $R \ll (\Omega/\omega_p^2)c$.⁸

From a theoretical point of view, the confinement problem for a nonneutral plasma is better defined, or simpler, than it is for a neutral plasma precisely because there are relatively few effects that can change P_θ . From an experimental point of view, careful precautions must be taken to minimize any external torques on the system.

Assuming that such torques can be made small enough and that the cylindrical wall is far enough out, the electrostatic interactions will have time to bring the electrons into thermal equilibrium with each other.⁹ Of course, the electrostatic interactions conserve P_θ . The one-electron thermal distribution is given by

$$f = n_0 \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \left[-\frac{1}{kT} (H - \omega p_\theta) \right], \quad (3)$$

where $H = mv^2/2 - e\phi(r, z)$ is the Hamiltonian for an electron and $p_\theta = mv_\theta - (e/c)A_\theta(r)r$ is the canonical angular momentum for an electron.¹⁰⁻¹² The parameters n_0 , T and ω are determined by the total number of electrons, energy and canonical angular momentum in the system. To see that the distribution corresponds to a set of confined electrons, note that the electric potential — i.e., $\phi(r, z)$ — forces the distribution to be exponentially small near the ends, where the negatively biased end cylinders are located, and that the vector potential forces the distribution to be exponentially small at large r , where the cylindrical wall is located.

Again, one can see the relative advantage in confining a nonneutral plasma rather than a neutral plasma. A neutral plasma which is confined by static electric and magnetic fields cannot be in thermal equilibrium. For example, in distribution (3) the sign of the charge enters in front of the scalar potential and in front of the vector potential; so confinement of electrons implies lack of confinement for ions. By and large, the notorious difficulty of confining and predicting the evolution of neutral plasmas derives from the fact that these plasmas are far from thermal equilibrium. One can argue that the existence of confined thermal-equilibrium states is an important property differentiating non-neutral plasmas from neutral plasmas. Incidentally, for the nonneutral case, one need not limit the discussion to *plasmas*. It has been suggested that a confined

pure electron plasma might be cooled to the liquid and crystal states.¹³ If the confined thermal equilibrium states can be realized in practice, by making the external torques negligible, a degree of experimental control and theoretical understanding may be achieved that is unprecedented in plasma physics. Also, the suggestion of cooling the electrons to the liquid and crystal states offers the possibility of exploring a large range of new and interesting physics.

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2. Dehmelt and F. Walls, Phys. Rev. Lett. 21, 127 (1968).
3. An electron trap of this kind, with radial confinement provided by an axial magnetic field and axial confinement provided by electrostatic fields, is generally referred to as a Penning trap. The experiments of Ref. 2 also employ a Penning trap.
4. R.C. Davidson, *Theory of Nonneutral Plasmas* (W.A. Benjamin, 1974) p. 4.
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7. This assumes that the fields due to the negatively biased end cylinders are not much larger than the self-fields.
8. To understand this, suppose that each electron experiences a radial displacement δr_j . The corresponding change in the canonical angular momentum is $\delta P_\theta \approx -m\Omega \sum_j r_j \delta r_j$ and the electrostatic energy liberated is $\delta U = \sum_j e E_r(r_j) \delta r_j \approx 2\pi n e^2 \sum_j r_j \delta r_j$; so one finds the relation $|\delta U| \lesssim (\omega_p^2/2\Omega) |\delta F_\theta|$. For an electromagnetic wave propagating down the cylinder beyond the end of the plasma, the energy and angular momentum satisfy the inequality $L \lesssim (R/c)W$. Thus, the energy liberated by the radial expansion is not adequate to supply the energy required by the wave

$$|\delta U| \lesssim \frac{\omega_p^2}{2\Omega} |\delta P_\theta| = \frac{\omega_p^2}{2\Omega} L \lesssim \frac{\omega_p^2}{2\Omega} \frac{R}{c} W \ll W \quad (3)$$

Electromagnetic radiation can remove only the small amount of angular momentum associated with the thermal motion of the electrons.

9. For the present experiments, the electrostatic interactions have time to establish a local thermal equilibrium (i.e., local Maxwellian), but not the global thermal equilibrium described by distribution (3).
10. R.C. Davidson, *Theory of Nonneutral Plasmas* (W.A. Benjamin, 1974) p. 10.
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