Three designs for a magnetic trap that will simultaneously confine neutral atoms and a non-neutral plasma

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Three trap designs are proposed for the simultaneous confinement of neutral atoms and a non-neutral plasma in close proximity. One design uses axially symmetric static magnetic fields with a magnetic minimum in a ring around the trap axis. Axial symmetry is required for confinement of the rotating non-neutral plasma, and the magnetic minimum traps the neutral atoms. The second design uses a rotating axially asymmetric magnetic field superimposed on a cusp field to create a time-averaged magnetic minimum (a ''TOP'' trap). The rotating asymmetry acts as a magnetic ''rotating wall'' to help confine the non-neutral plasma. In the third design, a cylindrically symmetric high-order multipole field traps the neutral atoms, which are made to rotate about the trap axis in order to avoid the magnetic null at the trap center. These designs may be useful for the production and confinement of cold antihydrogen.
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I. INTRODUCTION

Over the past decade, minimum B magnetic traps have developed into a standard technology for trapping neutral clouds of atoms.[1] These traps use magnetic fields with a local minimum in $|\mathbf{B}(\mathbf{r})|$ at a point where $|\mathbf{B}|$ remains nonzero.

One standard trap design, the ''Ioffe–Pritchard'' trap,[2] is shown schematically in Fig. 1. A pair of Helmholtz coils create a saddle point in $|\mathbf{B}|$ at the trap center. The saddle is transformed to a minimum by four Ioffe bars. The magnetic minimum creates a potential well for some atoms since atomic spins μ align either with or against the magnetic field. The magnetic dipole energy for spins aligned against the field is

$$E_M = -\mu \cdot \mathbf{B} = |\mu||\mathbf{B}|, \tag{1}$$

which is minimized at the trap center. The magnetic field does not vanish in the well; otherwise a spin could flip at the magnetic null (''Majorana flips''[3]) and the magnetic minimum would then expel the atoms rather than confine them.

The Ioffe–Pritchard trap is an excellent configuration for many purposes. However, the design is not cylindrically symmetric, and this is a problem for certain applications. One such application is the production and confinement of cold antihydrogen. Two experiments are currently underway to achieve this goal.[4] The experiments use Penning traps to trap two cold single species non-neutral plasmas consisting of anti-protons and positrons respectively, from which it is hoped that antihydrogen can be created by careful combination of the two species. The antihydrogen must then be confined in a neutral atom trap. A recent analysis[5] has shown that stable single-particle orbits exist for individual positrons and antiprotons trapped in the non-axisymmetric magnetic field produced by an Ioffe–Pritchard trap. However, if the trapped single-species plasmas are cold and of high density so as to maximize recombination rates when they are combined, the large-scale static field asymmetries produced by the Ioffe bars will degrade plasma confinement, causing plasma heating and charged particle loss at levels that are probably unacceptable.[6]

In this paper we consider three possible methods for trapping neutral atoms in close proximity to a cold, dense non-neutral plasma. The first method relies on a novel cylindrically symmetric magnetic configuration that can be used as a Penning trap to confine a non-neutral plasma, but also has a minimum in $|\mathbf{B}|$ so that it can simultaneously trap neutral atoms. The magnetic minimum is on a ring around the axis of symmetry of the trap. We will show that potential well depths of order 1 K or more should be achievable with this trap design. It is important for the well to be as deep as possible, since the antihydrogen will be created at the plasma temperature of a few K, and will also have kinetic energy associated with the plasma rotation.

A second design uses a standard ''time-averaged orbiting potential'' (TOP) trap[7] to confine the neutral atoms. Like the Ioffe–Pritchard trap, this design imposes large-scale azimuthal magnetic asymmetries, but the asymmetries are made to rotate around the trap axis. This rotating magnetic field has two effects: first, the torque exerted on the plasma by the rotating field may act to spin up the plasma, possibly even keeping it confined indefinitely in much the same manner as electrostatic ''rotating wall'' asymmetries used in other experiments.[8] Second, the rotating field, in concert with a static cusp field, creates a time-averaged magnetic minium at the trap center, with $|\mathbf{B}| \neq 0$ there. However, the effective depth of this potential minimum is probably limited to of order 0.1 K or less.

We also consider a third design that, like the first design, uses static cylindrically symmetric magnetic fields to confine

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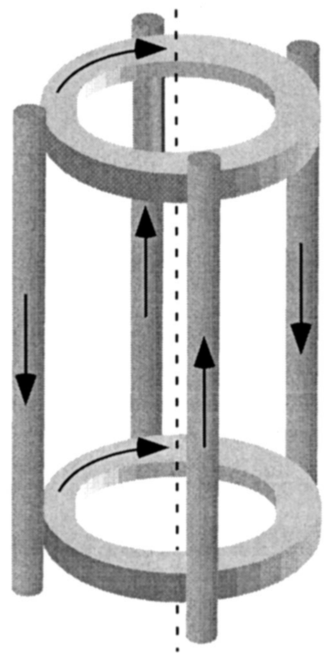

FIG. 1. Magnetic coil configuration of an Ioffe–Pritchard neutral atom trap. Arrows show the direction of currents.

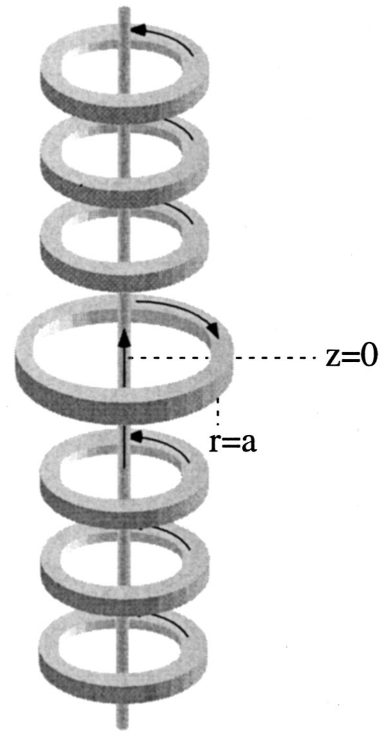

FIG. 2. Magnetic coil configuration of a cylindrically symmetric minimum-*B* Penning trap.

both the plasma and the neutral atoms. Here the magnetic minimum is a null at the trap center. In order to keep neutral atoms away from this magnetic null, the neutral atom cloud is made to spin, creating a centrifugal potential that repels the cloud of neutrals from the trap center. The rate of rotation may be controlled with a small rotating magnetic field. The main static magnetic field contains sufficiently high multipole moments $(n=4$ or higher) in order to overcome the centrifugal potential at large radius and trap the atoms in a ring. We will show that well depths of order 1 K or more could be obtained in this design, depending on the rotation frequency of the neutral atoms and the strength of the magnetic field.

Although antihydrogen formation requires the recombination of positron and antiproton plasmas, the work presented here considers only the trapping of a single species plasma in conjunction with a neutral atom trap. There are several reasons for this approach: first, separate single species plasmas must be confined and cooled before recombination can be attempted. It would be easiest to do this nearby to the region where recombination and neutral atom trapping will occur, so it is important to consider the equilibrium of single species plasmas in the fields created by a neutral atom trap. Second, the recombination process itself is not yet understood: the partially neutralized plasma may exhibit a host of instabilities, and issues regarding both the axial and radial confinement of such plasmas have not yet been resolved, although progress is being made.[9–11] By focussing on the trapping of a single species plasma in conjunction with neutral atoms, our work avoids these thorny issues.

However, our work could have some bearing on the recombination process: one possible recombination technique would be to allow only a relatively small number of antiprotons at a time to enter into the positron region, so that the positron plasma remains nearly completely unneutralized. It might be hoped that a small density of antiprotons would have an insignificant effect on the positron stability, and that a description of the system as a single species plasma might then be relevant. In this scenario, one of the traps discussed in this paper could serve as the recombination section.

As a final caveat, the magnet designs presented here have not been optimized with regard to engineering feasibility. Some of the designs are somewhat unusual, and may be difficult to realize in their present form. In particular, design limitations due to stresses on the magnetic coils will not be discussed. However, it is hoped that one or more of these concepts may prove to be a useful first step in the design of a realistic neutral atom/non-neutral plasma trap.

II. A CYLINDRICALLY SYMMETRIC MINIMUM-*B* PENNING TRAP

Cylindrically symmetric minimum-*B* equilibria have received considerable attention in the neutral plasma community due to their superior MHD stability properties.[12] Our first design is similar to the "stuffed cusp," but with an added solenoidal magnetic field, which is required for non-neutral plasma confinement.

A schematic of the magnetic elements in the trap is shown in Fig. 2. The trap consists of a solenoidal magnetic field $B_0\hat{z}$, which for simplicity is assumed to be uniform; an azimuthal magnetic field $B_\theta a\hat{\theta}/r$ created by a wire aligned along the axis of the solenoid, and the field from a current loop concentric with the solenoid whose current runs in the opposite direction to that of the solenoid. The latter field

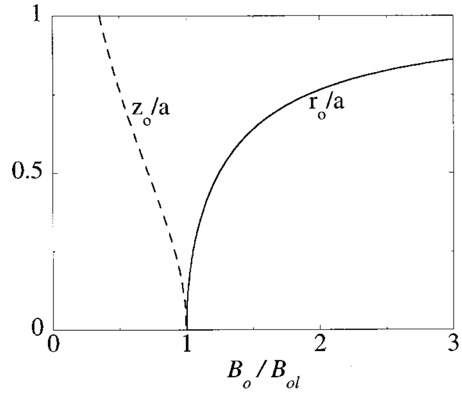

FIG. 3. r and z location of the field null as the ratio of the solenoidal to loop field is varied. No B_θ field is applied.

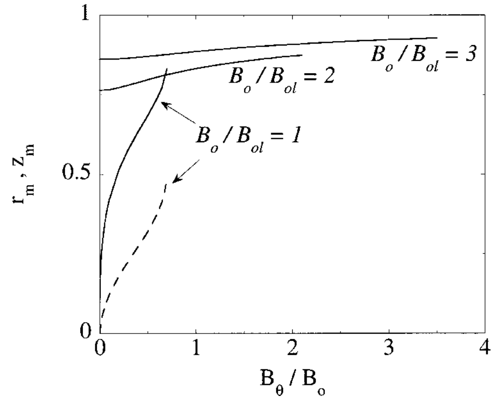

FIG. 4. Location of magnetic minimum versus azimuthal field B_θ for three values of the ratio of solenoidal to loop fields, B_0/B_{0_ℓ}. Solid curves are radial position, dashed curve is the axial position for $B_0/B_{0_\ell}=1$.

$\mathbf{B}_\ell(r,z)$ is most easily written in terms of the vector potential associated with the loop, assumed to have radius a,

$$\mathbf{B}_\ell(r,z)=-\frac{\partial A_{\theta_\ell}}{\partial z}\hat{r}+\frac{1}{r}\frac{\partial}{\partial r}(rA_{\theta_\ell})\hat{z}, \tag{2}$$

where

$$A_{\theta_\ell}(r,z) =-\frac{2B_{0_\ell}a^2}{\pi\sqrt{a^2+r^2+z^2+2ar}}\left[\frac{(2-m)K(m)-2E(m)}{m}\right], \tag{3}$$

$$m=\frac{4ar}{a^2+r^2+z^2+2ar}, \tag{4}$$

$K(m)$ and $E(m)$ are the complete elliptic functions,[13] and $B_{0_\ell}=B_l(r=0,z=0)$.

When $B_{0_\ell}<B_0$, the loop magnetic field cancels out the solenoidal field at a ring of radius r_0 in the plane of the loop (the $x-y$ plane). When $B_{0_\ell}>B_0$, the field is cancelled at points $\pm z_0$ along the z axis (see Fig. 3).

Without an applied azimuthal field B_θ, the ring at $r=r_0$ and $z=z_0$ is both a magnetic mimimum and a magnetic null, which makes it unusable as an atom trap. With an azimuthal field, $|\mathbf{B}|$ no longer vanishes. Since the azimuthal field is monotonically decreasing with r, the location of the minimum is shifted outwards to a location $r_m>r_0$. For B_θ sufficiently large, the minimum disappears as it is pushed toward the loop. The dependence of r_m on B_θ/B_0 is shown in Fig. 4 for several values of B_{0_ℓ}/B_0. When $B_{0_\ell}/B_0\geqslant 1$, the minimum location is at finite $z=\pm z_m$. The variation of z_m with B_θ/B_0 is also shown in Fig. 4 for one choice of B_0/B_{0_ℓ}.

Contours of constant $|\mathbf{B}|$ are shown in grayscale in Fig. 5 for the choices $B_0/B_{0_\ell}=2.6$, giving $r_0=0.81a$; and $B_\theta/B_0=0.125$, giving $r_m=0.83a$. For these parameters, contours of constant $|\mathbf{B}|$ with $|\mathbf{B}|/B_0\lesssim 0.9$ do not intersect the walls of the apparatus.

For antihydrogen in the ground state, $|\mu|\simeq\hbar e/2m_e c$. According to Eq. (1), a solenoidal field of $B_0=2$ T therefore corresponds to a potential well of depth $(0.9-0.125/0.83)\mu B_0=1$ K. Atoms with temperature less than this should collect around the minimum at $r=r_m$. A field $B_0\gtrsim 8$ T increases the well depth above the temperature of liquid helium, obviating the need to cool the trap walls with a dilution refrigerator.

We now turn to the non-neutral plasma confinement characteristics of this trap design. It is well known that single-species plasmas can be confined in a thermal equilibrium state by static cylindrically symmetric fields.[14] The unneutralized charge cloud rotates about the trap axis with rotation frequency ω. For low plasma temperature such that the Debye length is small compared to the plasma size, the plasma density in thermal equilibrium is determined by the equation[14]

$$n(r,z)=\frac{m\omega}{2\pi e^2}(\Omega_{c_z}(r,z)-\omega), \tag{5}$$

where $\Omega_{c_z}=eB_z(r,z)/mc$ is the cyclotron frequency based only on the axial component of the total magnetic field. Here e is the charge and m the mass of the plasma particles. Note that the azimuthal field B_θ does not play a role in non-neutral plasma confinement.

Equation (5) follows from the fact that rotation through a magnetic field creates an effective potential well $\phi_B(r,z)$ for the particles, where

$$\phi_B(r,z)=\frac{e\omega r}{c}A_\theta(r,z)-\frac{m\omega^2r^2}{2}. \tag{6}$$

The second term is the deconfining centrifugal potential and $A_\theta(r,z)$ is the θ-component of the magnetic vector potential, given by

$$A_\theta(r,z)=A_{\theta_\ell}(r,z)+\tfrac{1}{2}B_0r^2, \tag{7}$$

where A_{θ_ℓ} is the loop vector potential, given by Eq. (3). The potential ϕ_B can be thought of as due to a fictitious neutralizing background charge of density $n(r,z)$; that is, $\nabla^2\phi_B(r,z)=4\pi e^2 n(r,z)$. The equilibrium plasma density matches the density n out to a surface of revolution where the supply of plasma charge is exhausted. The shape of this surface is determined by the condition that it is an equipotential in the frame rotating with the plasma. Therefore to find the plasma shape, we solve the equation

$$\phi(r,z)|_S=\text{constant}, \tag{8}$$

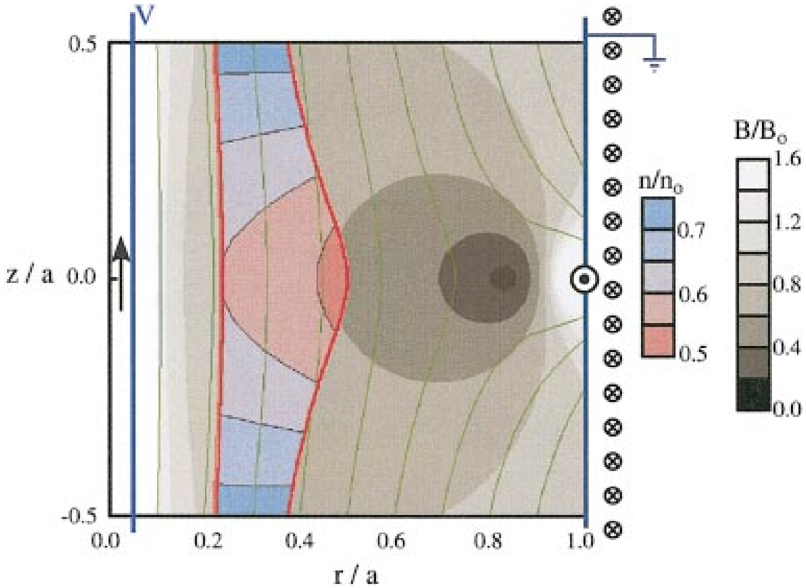

FIG. 5. (Color) An example of plasma confinement in the cylindrically symmetric minimum-B Penning trap. Plasma density is shown by colored contours, magnetic field intensity is shown by gray scale contours. Green lines are magnetic flux surfaces. Arrows denote the location of current-carrying wires. Thick blue lines are electrodes. In this example, $B_0/B_{0_\ell}=2.6$, $B_\theta/B_0=0.125$, and $V=0.38\pi e n_0 a^2$.

where S is the surface of the plasma, and ϕ is the total potential, including the self-consistent potential $\phi_P(r,z)$ from the plasma itself, and the potential $\phi_V(r,z)$ from voltages on surrounding electrodes:

$$\phi(r,z)=\phi_P(r,z)+\phi_V(r,z)+\phi_B(r,z). \tag{9}$$

Since our design has a central wire, required for the azimuthal field B_θ, a voltage V must be applied to the wire in order to repel plasma charges. This creates a hollow plasma. In Fig. 5 we assume that the wire has radius $b=0.05a$, and that there is a surrounding grounded cylindrical electrode at $r=a$. The plasma is assumed to be a long column. For large $|z|$ away from the loop, the magnetic field is nearly uniform, and the plasma forms a hollow column with uniform density $n_0=(m\omega/2\pi e^2)(\Omega_0-\omega)$, where $\Omega_0=(eB_0/mc)$ is the cyclotron frequency associated with the solenoidal field only. The inner and outer radii of the hollow column, r_1 and r_2, are related to the voltage V on the inner wire by the condition that $\phi(r_1,z)=\phi(r_2,z)=$constant:

$$V=\pi e n_0\left[r_2^2-r_1^2+2r_1^2\ln\left(\frac{r_1}{b}\right)-2r_2^2\ln\left(\frac{r_2}{a}\right)\right]. \tag{10}$$

In Fig. 5, we have chosen $r_1=0.2a$, $r_2=0.3a$, so that $V=0.38\pi e n_0 a^2$. We have also assumed that $\omega/\Omega_0\ll 1$, so that we can neglect the centrifugal potential in ϕ_B [see Eq. (6)]. In this case we need not specify ω in determining the plasma equilibrium.

One can see from the figure that the plasma expands radially and decreases in density near $z=0$, since the confining B_z field is weakest here. One can also see that the minimum B ring is well outside the plasma. This is hardly surprising, since B_z vanishes at $r_0<r_m$, and the plasma must be confined in a region away from this point according to Eq. (5). In principle, it is possible to construct finite-length plasma equilibria that contain the minimum ring at r_m since B_z does not vanish at r_m, but rather at r_0. However, the confinement potential $\phi(r,z)$ exhibits only a very weak minimum in this case.

Since the plasma rotates with frequency ω, neutral atoms created by recombination at radius r within this plasma will be created with a certain amount of angular momentum, $\ell_z=Mv_\theta r$, where M is the atomic mass and v_θ is the azimuthal velocity, which has an average value of ωr. This will create a tendency for the neutral atom cloud to spin. This rotation creates a deconfining centrifugal potential for the neutral atoms, which can be expressed as an additional term in Eq. (1):

$$E_M=|\mu||\mathbf{B}|+\frac{1}{2}\frac{\ell_z^2}{Mr^2}. \tag{11}$$

So far we have assumed that this centrifugal term is negligible. However, this depends on the size of ℓ_z. In typical experiments, the rotation frequency $\omega/2\pi$ is on the order of several kHz. Taking the plasma radius to be $r_p\simeq 1$ cm, we may estimate angular momentum due to rotation as $\ell_z\sim M\omega r_p^2$. Then for an antihydrogen atom the centrifugal term is

$$0.5M\omega^2 r_p^4/r^2=0.24\,\mathrm{K}\left(\frac{\omega/2\pi}{1\,\mathrm{kHz}}\right)^2\left(\frac{r_p}{r}\right)^2\left(\frac{r_p}{1\,\mathrm{cm}}\right)^2.$$

One can see that a rotation frequency of 1 kHz and a plasma radius of 1 cm leads to a small centrifugal correction to E_M; however, $\omega/2\pi=10$ kHz would cause a large change in the magnetic well, possibly leading to deconfinement, depending on the ratio of r_p/r at the location of minimum B. In order to trap large densities required for rapid recombination, a large magnetic field will therefore be required so that ω remains small.

Another potential difficulty with this design involves the central wire. Astute readers may already have noted that producing a tesla size field with a central wire (or wires) requires exceedingly high currents. Fortunately, the azimuthal field produced by the central wires need not be this large. Recall that the only purpose of the azimuthal field is to prevent Majorana flips by keeping $|\mathbf{B}|$ finite at the magnetic

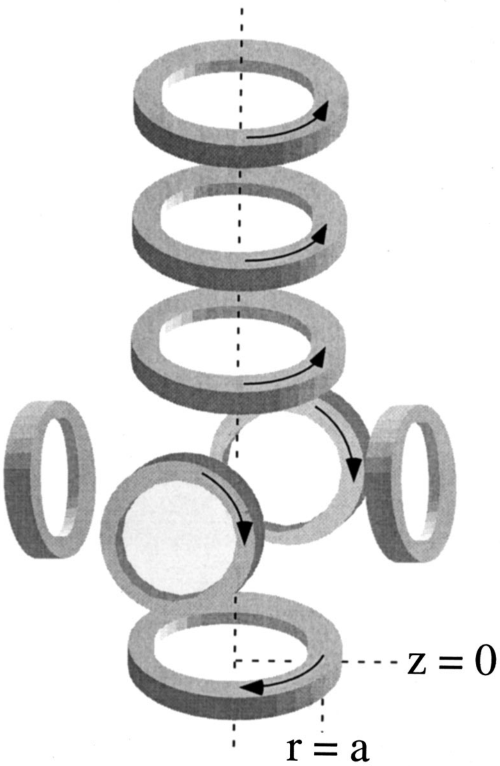

FIG. 6. Schematic of the magnetic coils for a TOP Penning trap.

minimum. For cryogenic atoms, this can be accomplished with modest azimuthal fields, on the order of a few gauss. The argument is as follows. Majorana flips are prevented when the minimum spin precession frequency, $2\mu B_{\min}/\hbar$ [where $B_{\min}=B(r_m,z_m)$ is the minimum magnetic field strength], is much greater than the maximum rate of variation of the magnetic field as seen by an atom moving through the minimum, $(\bar{v}\boldsymbol{\nabla}B/B)_{\max}\sim\bar{v}B_0/aB_{\min}$, where $\bar{v}$ is the thermal speed of the atoms. Comparing these two rates for antihydrogen yields

$$B_{\min}\geqslant 1.3\ \text{Gauss}(B_0/1\ \text{Tesla})^{1/2}(T/1\ \text{K})^{1/2}(a/1\ \text{cm})^{-1/2}. \tag{12}$$

Thus, a field of 10–100 Gauss at the magnetic minimum should be sufficient to prevent any Majorana flips from occurring, provided that the atoms are cold.

Finally, we note that it is possible to create a ring-minimum-B configuration that does not need a central wire: the "Furth–Andreoletti" trap.[12] This considerably simplifies the trap design, but unfortunately the depth of the magnetic minimum in such traps is exceedingly weak.[15] In Sec. IV we will discuss an axisymmetric trap design that also avoids the central wire, but has a well depth of order 1 K or more.

III. "TOP" PENNING TRAP

This design uses a standard time-averaged orbiting potential (TOP) trap, combined with a Penning trap, as displayed schematically in Fig. 6. The non-neutral plasma is confined in the upper section by a solenoidal magnetic field. In the lower section this field is cancelled by an opposing field, creating a cusp with a null on the z axis, below the

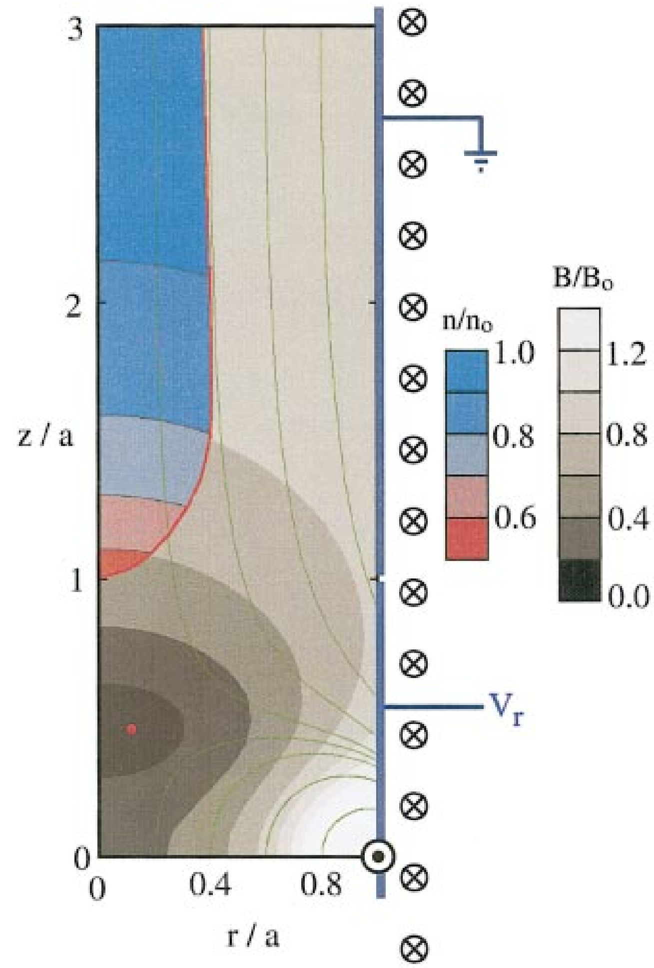

FIG. 7. (Color) An example of plasma confinement in a TOP Penning trap. Plasma density is shown by colored contours, time-averaged magnetic field intensity $\langle|\mathbf{B}_t|\rangle$ is shown by gray-scale contours. Green lines are magnetic flux surfaces neglecting the rotating field. Arrows in and out of the page denote the location of current-carrying wires. Thick blue lines are electrodes. Red dot is the location of the "circle of death." In this example $B_0/B_{0\not}=0.75$, $B_r/B_0=0.05$, and $V_r=0.6\pi e n_0 a^2$.

plasma. Neutral atoms are trapped in the magnetic well created by the cusp. The plasma is held away from the null by a biased cylindrical electrode (not shown).

To this configuration is added a rotating magnetic field, predominantly in the $x-y$ plane, rotating with frequency ω. The field is created by $m=2$ (or more) pairs of Helmholtz coils located about the trap as shown. The current in the coils is oscillated at frequency ω/m, and phased by π/m between adjacent pairs of coils.

For simplicity in the following analysis, we take the solenoidal field and rotating field to be uniform, and the cusp to be created by a single current loop, resulting in

$$\mathbf{B}_t=\mathbf{B}_{\not}(r,z)+B_0\hat{z}+B_r(\hat{x}\cos\omega t+\hat{y}\sin\omega t), \tag{13}$$

where B_r is the strength of the rotating field, and ω is its rotation frequency. This frequency must be chosen to be small compared to the spin precession frequency $2\mu|B_t|/\hbar$ in order for the spins to be able to follow the rotating field without flipping; but the rotation must be rapid compared to the dynamical frequencies of the antihydrogen in the trap. If so, the rotating field can be time averaged, resulting in an average magnetic potential energy

$$E_{\mathrm{TOP}}=\mu\langle|\mathbf{B}_t|\rangle=\mu\frac{2\pi}{\omega}\int_0^{2\pi/\omega}dt|\mathbf{B}_t|. \qquad (14)$$

Near the cusp, located at $z=z_c$, a Taylor expansion of $\mathbf{B}_{\ell}$ yields

$$\mathbf{B}_{\ell}\approx B'(r\hat{r}-2(z-z_c)\hat{z})-B_0\hat{z}, \qquad (15)$$

where B' is the radial derivative of $\mathbf{B}_{\ell}$ at the cusp; $B'=\partial B_{\ell_r}/\partial r|_{r=0}^{z=z_c}$. The average over $|\mathbf{B}_t|$ can then be easily performed, yielding

$$E_{\mathrm{TOP}}=\frac{a}{2}(r^2+2(z-z_c)^2), \qquad (16)$$

where $a=B'^2/(2B_r)$. Thus, the averaged potential E_{TOP} exhibits a minimum at the location of the cusp; but the magnetic field itself is nonzero there, taking on the value B_r.

Plasma confinement in this setup is similar to the electrostatic ''rotating wall'' perturbations used in several non-neutral plasma experiments, where the rotating perturbation is created by oscillating voltages applied to $2m$ sectors centered at $\theta_j=\pi j/m$.[8] It has been shown that electrostatic wall perturbations with appropriate phase can be used to confine a plasma indefinitely, over a range of temperatures from mK to eV.

It is currently unknown whether a rotating magnetic field will work as a magnetic ''rotating wall,'' because there have been no experiments to test this concept. However, there has been considerable work on a similar configuration, the ''rotamak,'' for use in driving currents in a neutral plasma. In this device fields of order 600 Gauss were made to rotate at 670 kHz, which should be sufficient for magnetic rotating wall confinement of non-neutral plasmas.[16] A rotating field in the $x-y$ plane causes the total field confining the non-neutral plasma to tilt slightly, and this tilt precesses around the z axis at frequency ω; ideally, this will spin up the plasma to frequency ω. For a 4 Tesla solenoidal field and a 600 Gauss rotating field the tilt is 0.9°. A static field tilted by this amount would cause rapid plasma loss,[17] indicating that such a rotating tilt would probably couple well to the plasma.

An example of a cold trapped plasma confined in this configuration is shown in Fig. 7. In this figure the effect on the plasma of the rotating magnetic field is neglected for simplicity: it would slightly distort the plasma equilibrium into a rotating 3D figure without cylindrical symmetry. The gray contours show the time-averaged value of $|B_t|$, compared to B_0, assuming that $B_r=0.05B_0$. We choose $B_0/B_{0_\ell}=0.75$, so that the cusp is at $z_c/a=\sqrt{(B_{0_\ell}/B_0)^{2/3}-1}=0.46$. The plasma is held away from this cusp by a single cylindrical electrode, biased to the voltage $V_r=0.6\pi e n_0 a^2$.

Note that at $B_r=600\,\mathrm{G}$, the spin precession frequency is 10^{10} rad/s, which is much larger than ω. Therefore, the atomic spins should easily be able to follow the rotating field without flipping and deconfining the atoms. Also, note that at 100 kHz, the skin depth of copper is roughly 200 μm. This may necessitate the use of nonconductive cylindrical electrodes with a thin conductive inner coating in order to allow the rotating wall field to penetrate into the plasma confinement region. Alternatively, an open design with large gaps between the electrodes could be attempted.

However, a 600 G rotating field may be insufficient for confinement of antihydrogen. This is because in a TOP trap, the magnetic field strength $|\mathbf{B}|$ goes to zero at a point that rotates around the trap center (termed the ''circle of death''). According to Eq. (13), the radius and height (r_d,z_d) of this circle is given by the solution to the coupled equations $B_{\ell_r}(r_d,z_d)-B_r=0$, and $B_{\ell_z}(r_d,z_d)+B_0=0$. Using Eq. (15), one obtains $z_d\simeq z_c$, and $r_d\simeq B_r/B'$. The location of this point is shown as a red dot in Fig. 7, for the given parameters of the figure.

To keep from suffering Majorana flips, neutral atoms must be confined within this circle, imposing an effective well depth of

$$E_{\max}=\mu\frac{B_r}{4}, \qquad (17)$$

according to Eq. (16). A 600 G rotating field yields $E_{\max}=20\,\mathrm{mK}$, which is more than sufficient for laser-cooled atoms but may be difficult to achieve with antihydrogen, for which efficient laser cooling techniques are still a subject of ongoing research.[18] While temperatures on this order (or even lower) have been achieved in magnetically trapped hydrogen using dilution refrigerators[19] and evaporative cooling,[20] such techniques may not be applicable to antihydrogen studies. Evaporative cooling throws away a large fraction of the atoms in order to lower the energy of the remainder, which may be problematic when dealing with small quantities of antihydrogen. Also, the thermal coupling of dilution-refrigerated walls to neutral antihydrogen atoms in a high-vacuum environment has not been clarified. In any case, the deeper the trap well, the better the chances of success.

By lowering the rotation frequency ω from hundreds to tens of kHz, larger rotating fields could be possible, possibly up to a few thousand Gauss. However, according to Eq. (17), even these large fields limit the maximum confinement temperature to of order 0.1 K. It may be possible to increase the potential well depth further by employing more complex rotating fields that include a quadrupole component.[21] However, the feasibility of rotating a quadrupole field of several thousand Gauss at tens of kilohertz is unclear.

If and when experimental techniques are developed so that milliKelvin temperature antihydrogen can be easily created, a design based on the TOP rotating magnetic field trap would probably be the most straightforward approach to confining both neutral antihydrogen and a non-neutral plasma. Until that time, however, the field geometry of Sec. II provides the required well depth of 1 K or more, as well as the cylindrical symmetry needed for plasma confinement.

IV. CYLINDRICALLY SYMMETRIC ROTATING ATOM TRAP

The configuration of magnets discussed in Sec. II includes a central current-carrying wire, which complicates the

TABLE I. Parameters of a cylindrically symmetric rotating neutral atom trap with a single multipole field component.

Multipole moment n	A_n	Radius of minimum r_m/a	Minimum energy $E_{B_{\min}}/\mu B_n$	Scaled maximum (rotation frequency)^2 $\alpha_{\max}$	Radius of minimum at $\alpha=\alpha_{\max}$ $(r_m/a)_{\max}$
4	3/2	$4\alpha/9$	$-2/9$ $(-.222)$	3/2	0.666 . . .
5	15/8	$(4\alpha/15)^{1/2}$	$-15/32$ $(-.469)$	15/8	0.7070 . . .
6	15/8	$(16\alpha/75)^{1/3}$	$-9/3200^{1/3}$ $(-.611)$	15/8	0.737 . . .
7	35/16	$(16\alpha/105)^{1/4}$	$-35/24\sqrt{3}$ $(-.842)$	35/16	0.760 . . .
8	35/16	$(32\alpha/245)^{1/5}$	$-25/8\cdot 392^{1/5}$ $(-.947)$	35/16	0.778 . . .
9	315/128	$(32\alpha/315)^{1/6}$	$-945/512\cdot 2^{2/3}$ (-1.16)	315/128	0.794 . . .

design. In this section, we discuss one possible alternative to the design of Sec. II which retains cylindrical symmetry, but has no central wire.

As discussed in Sec. II, rotation of the neutral atoms about the trap axis creates a deconfining centrifugal potential. For single particles, the effective potential is given by Eq. (11). This potential repels any atom with $\ell_z \neq 0$ away from the trap center. Therefore, if one creates a magnetic null at $\mathbf{r}=0$ to trap the neutral atoms, and if one can cause the neutral atoms to rotate about the trap axis so that all atoms have $\ell_z \neq 0$, the atoms can be trapped in the magnetic well but will avoid the central magnetic null. Furthermore, since the atoms will be created through recombination in a rotating plasma they will already have an angular momentum associated with this rotation. The confinement scheme would be similar to that displayed in Fig. 6; the non-neutral plasma would be confined in the upper section, and the neutral atoms would be confined in the lower section (using a cylindrically symmetric configuration of magnets, to be described presently).

Even if the bulk rotation of the neutral cloud is maintained, thermalization of the neutrals through atom–atom collisions would eventually create atoms with sufficiently low ℓ_z such that these atoms would fall through the null at the trap center and be lost due to Majorana flips. This loss process can be avoided through careful design of the magnetic well and careful control of the neutral rotation frequency, as we will now show.

For a thermalized gas of neutral atoms, the single-particle potential of Eq. (11) should be replaced by the Boltzmann potential associated with gas rotating with uniform rotation frequency ω:

$$E_B=\mu B-\tfrac{1}{2}M\omega^2 r^2. \tag{18}$$

This potential energy enters in the Boltzmann thermal distribution,[22]

$$f(\mathbf{r},\mathbf{v})=(2\pi k_B T/M)^{-3/2} n_p e^{-M(\mathbf{v}-\omega r\hat{\theta})^2/2k_B T} e^{-E_B(r,z)/k_B T}, \tag{19}$$

where n_p is a constant (the neutral particle density at the trap center where $E_B=0$). The centrifugal term in Eq. (18) will still repel particles from the trap axis, provided that the attractive magnetic potential μB is sufficiently weak near the axis. This can be arranged by employing a high order multipole field. Assume for the moment that $\mathbf{B}$ has only a single multipole component, that is,

$$\mathbf{B}=\nabla\left(B_n\left(\frac{r_s}{a}\right)^n P_n(\cos\theta)\right), \tag{20}$$

where B_n is a constant, r_s is spherical radius and θ is the usual spherical polar coordinate.

Then one can see that $n\geqslant 4$ is required, so that $|\mathbf{B}| \propto r_s^{n-1}$ is sufficiently weak at small r_s and the centrifugal force creates a local maximum of E_B at $r=0$. Also, these higher multipoles are sufficiently strong at larger r_s to overcome the centrifugal force and confine the atoms in a circulating ring about the trap axis. Atoms will then be unable to reach the origin provided that the well depth is larger than $k_B T$.

For $n\geqslant 4$ there is a minimum in E_B in the $z=0$ plane, at finite r. The energy in the $z=0$ plane can be written, using Eqs. (18) and (20), as

$$E_B(r,z=0)=\mu B_n\left[A_n\left(\frac{r}{a}\right)^{n-1}-\alpha\left(\frac{r}{a}\right)^2\right], \tag{21}$$

where $A_n=\sqrt{n^2 P_n^2(0)+P_n'^2(0)}$, the prime indicates differentiation, and $\alpha=M\omega^2 a^2/(2\mu B_n)$ is a dimensionless number parameterizing the relative strength of the centrifugal and magnetic forces. For $n\geqslant 4$, Eq. (21) has a minimum at $r=r_m=a(2\alpha/(n-1)A_n)^{1/(n-3)}$. At the minimum, E_B takes on the value

$$E_{B_{\min}}=-\mu B_n\alpha\left[\frac{2\alpha}{(n-1)A_n}\right]^{2/(n-3)}\frac{n-3}{n-1}, \tag{22}$$

which shows that as α increases, the minimum energy becomes more negative and the well becomes deeper. However, Eq. (21) implies that the largest well depth occurs at $\alpha=\alpha_{\max}=A_n$; otherwise $E_B(r=a,z=0)<0$, and according to Eq. (19) it begins to be more likely for particles to hit the magnetic coils at $r=a$ than to escape to the origin. The values of $\alpha_{\max}$ and corresponding values of $|E_{B_{\min}}|$ and r_m are given in Table I. Also, $E_B(r,z=0)$ is displayed in Fig. 8 for several values of n and for $\alpha=\alpha_{\max}$.

One can see from the table and the figure that the well depth can meet or even exceed μB_n, depending on the values of n and α. Furthermore, the higher the order of the multipole, the deeper the confinement (although at high n the radius of the minimum approaches a). Values of B_n of a Tesla or more should be technically feasible, allowing trap depths of order 1 K or more. For example, if we take $n=6$

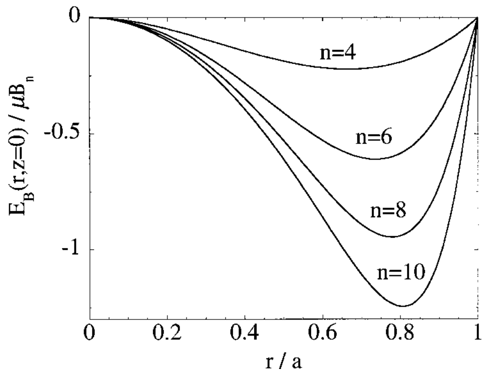

FIG. 8. Boltzmann potential vs radius r in the $z=0$ plane for several perfect multipole fields, $n=4$, 6, 8, and 10. The potential is shown for the case of maximum well depth, where scaled rotation frequency $\alpha=\alpha_{\max}$, so that $E_B=0$ at $r=a$.

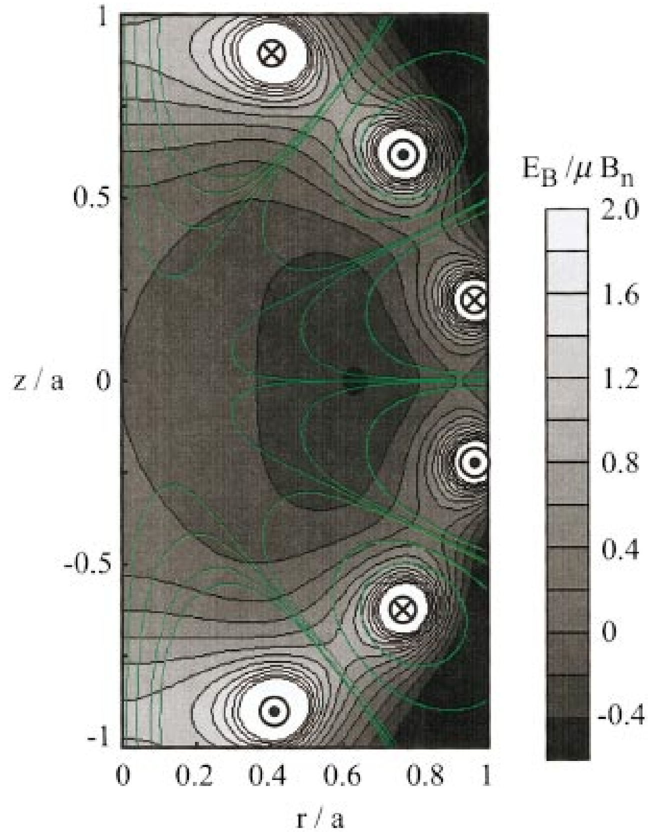

FIG. 9. (Color) The Boltzmann potential from an approximate $n=6$ multipole, created by 6 current loops. Gray contours are contours of constant $E_B/\mu B_6$, for maximum well depth ($\alpha=0.85$ chosen such that $E_B=0$ at $r=a$). Green lines are magnetic field lines.

and $B_6=5$ Tesla, then Table I implies the maximum well depth is $|E_{B_{\min}}|=2$ K, realized at a rotation frequency of $\omega=32\,\text{kHz}\times 1/a(\text{cm})$.

The magnetic field does not need to be a perfect multipole for the design to work. Consider, for example, the field created by 6 current loops on the surface of a sphere of radius a, separated in θ by equal amounts, and given alternating currents of the correct magnitude to null out B_z, $\partial^2 B_z/\partial z^2$ and $\partial^4 B_z/\partial z^4$ at the origin. (The first and third derivatives of B_z vanish by symmetry of the configuration.) The field is of the form of Eq. (20) near the origin with $n=6$, but differs substantially from Eq. (20) as $r\rightarrow a$. Nevertheless, a contour plot of E_B vs r and z for this configuration of currents, shown in Fig. 9 (for the case of maximum well depth), shows that the confinement is nearly as good as for a perfect $n=6$ multipole, both in terms of maximum well depth and spatial extent of the well.

For any cryogenic system of particles rotating at tens of kilohertz, heating due to conversion of rotational energy into thermal energy is clearly an important concern. Unfortunately, not much is known about this phenomenon in neutral atom traps. Although experiments have been performed on rotating Bose–Einstein condensates, rotation rates of only a few hundred hertz were obtained by torquing on the atoms with lasers,[23] and in these experiments the heating due to rotation was apparently not an important problem. Nevertheless, several possible mechanisms present themselves for heating the atoms by slowing their rotation. One obvious candidate, collisions with background neutrals, can presumably be minimized by operating in a cryogenic high vacuum environment. Another possible mechanism is the scattering of the circulating atoms by static magnetic field asymmetries. Such asymmetries always exist at some level, even in the most carefully constructed traps, and would eventually lead to slowing of the rotational motion, accompanied by heating and particle loss.

One possible method of circumventing this problem would be to again apply a magnetic "rotating wall." This time, the purpose is to keep the neutrals rotating, although it could also be used to control the plasma. Since the rotational drag from magnetic field errors and/or other processes is presumably weak, the magnitude of the rotating field could probably be considerably smaller than the 600 Gauss fields discussed in the previous section. The torque from this rotating field (and from the static field errors) is found using Eq. (1):

$$\tau=-\mu\frac{\partial B}{\partial\theta}. \tag{23}$$

This equation shows that even a simple uniform rotating field, as in Eq. (13), could work to torque on the atoms and maintain their rotation at some equilibrium level. By slowly varying the rotation rate of the field, the atoms could be accelerated or decelerated as their rotation rate equilibrates with that of the field. This method has proven to be quite successful when applied (using electrostatic fields) to nonneutral plasmas rotating at up to hundreds of kilohertz.[8] However, it is currently unknown whether the rotating field technique will work with neutral atoms.

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