Hollow electron column from an equipotential cathode

C. F. Driscoll and J. H. Malmberg

Department of Physics, University of California, San Diego, La Jolla, California 92037
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An unneutralized, magnetically confined electron column thermally emitted from an equipotential cathode is considered. An analytic, scaled solution of the radial Poisson's equation indicates that the column is hollow and less than $\sqrt{8}$ central Debye lengths in radius.

The possibility of using nonneutral plasmas as an experimentally accessible means of studying general plasma properties has been suggested and pursued.[1,2] Davidson[1] has given dynamic equilibrium distribution functions for plasma columns, independent of source considerations. Here, we consider an unneutralized thermal column formed by an equipotential cathode and derive the radial density and potential profiles. We find that the space charge potential limits the central density which can be obtained, resulting in a column which is at most a few central Debye lengths in radius.

Consider the physical configuration shown in Fig. 1. A uniform Maxwellian distribution of electrons is emitted from a grounded thermionic disc cathode at temperature T. The electrons drift along an essentially infinite magnetic field $B_0\hat{z}$ coaxial with a grounded conducting cylinder centered around the cathode. The far end plate (anode) of the cylinder is biased substantially more negative than any interior potential. After a z-dependent region near the cathode, the axially symmetric, z-independent potential $V(r)$ is given by Poisson's equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)=4\pi e n(r)$$

with

$$\left.\frac{\partial V}{\partial r}\right|_{r=0}=0, \quad V(R_w)=0,$$

where $n(r)$ is the density of electrons as a function of radius, $-e$ is the electron charge, and R_w is the radius of the conducting cylindrical wall. The charge density is in turn determined by the potential. We assume a steady-state potential which decreases monotonically in z from the grounded cathode to the negative end plate. In this case, the distribution of electron velocities is everywhere a Maxwellian, symmetric in the $\pm v_z$ directions (each electron with $+v_z$ is eventually reflected to the same position with $-v_z$). Those emitted electrons energetically able to reach a potential V form a density proportional to $\exp(eV/kT)$, where k is Boltzmann's constant. The density as a function of radius in the z-independent region can then be expressed as

$$n(r)=n(0)\exp\left\{\frac{e}{kT}\left[V(r)-V(0)\right]\right\}. \tag{1}$$

This configuration differs from thermal beam studies such as Langmuir's[3] in that the axial potential is not fixed, and electrons do not move radially.

Combining the Poisson and density equations, using a scaled potential Φ, and scaling the radius in terms of the central Debye length gives

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial \Phi}{\partial \rho}\right)=\exp[\Phi(\rho)], \tag{2}$$

where

$$\Phi(\rho)\equiv\frac{e}{kT}[V(\rho)-V(0)], \quad \rho\equiv r/\lambda(0),$$

$$\lambda(r)\equiv[kT/4\pi e^2 n(r)]^{1/2}.$$

A solution can be given which satisfies the boundary condition of zero electric field ($\partial\Phi/\partial\rho=0$) at $\rho=0$, as well as the identity $\Phi(0)=0$. The potential inside the plasma boundary ρ_p is

$$\Phi(\rho)=-2\ln(1-\rho^2/8), \quad \rho<\rho_p. \tag{3}$$

Note that the parameter $n(0)$ appears only indirectly through the scaled radius ρ; solutions for different central densities follow the same solution with different radius scales. The corresponding solution for the density is

$$n(\rho)=n(0)\,(1-\rho^2/8)^{-2}. \tag{4}$$

These solutions are displayed in Fig. 2. If the cathode does not fill the cylinder radially, the potential solution can be extended outside the plasma by integrating Laplace's equation and matching the solutions at the plasma boundary. This gives

$$\Phi(\rho)=\frac{\rho_p^2/2}{1-\rho_p^2/8}\ln\left(\frac{\rho}{\rho_p}\right)-2\ln\left(1-\frac{\rho_p^2}{8}\right), \quad \rho_p<\rho<\rho_w. \tag{5}$$

The unscaled potential V can be recovered as $V(r)=(kT/e)\{\Phi[r/\lambda(0)]-\Phi[R_w/\lambda(0)]\}$. In the limit of very low cathode emission, $n(0)\to 0$, $\lambda(0)\to\infty$, and $\rho_p\to 0$ for any given cathode radius R_p. The potential $\Phi(\rho)$ is then approximately zero, and the density is approximately $n(0)$ for all radii $\rho<\rho_p$. As the cathode emission is increased, ρ_p will become significant compared with $\sqrt{8}$ and some of the structure of Eqs. (3) and (4) will be seen in the potential and density.

Some simple conclusions follow from this solution. It is apparent that the plasma must be less than $\sqrt{8}$

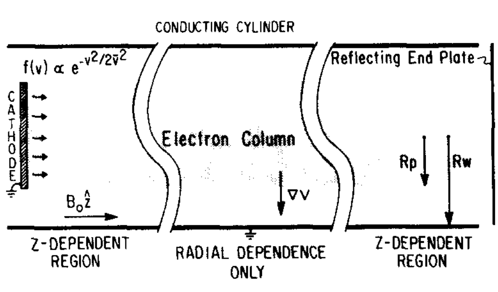

FIG. 1. Cross section of the cylindrical configuration.

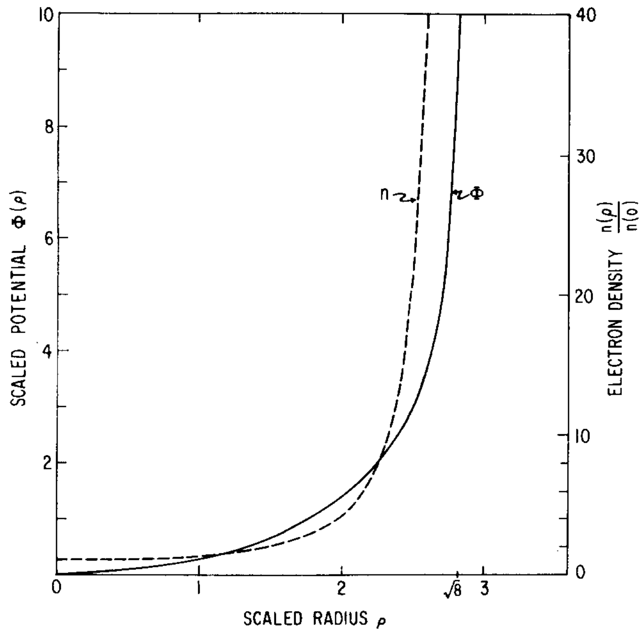

FIG. 2. Scaled potential and normalized density vs scaled radius.

central Debye lengths in radial extent; a column with $\rho_p = \sqrt{8}$ would have an infinite potential difference between its center and edge. This condition can be restated as a limit to the attainable central density for any given cathode (and plasma) temperature and radius. Since $R_p[kT/4\pi e^2 n(0)]^{-1/2} \le \sqrt{8}$, numeric substitution gives

$$n(0) \le 4.42 \times 10^6 \frac{kT(\mathrm{eV})}{R_p^2(\mathrm{cm}^2)}\ \mathrm{cm}^{-3}.$$

As more electrons are emitted by the cathode, the central potential becomes more negative, and $n(0)$ asymptotes to the above value; the edge density, however, increases without bound as the cathode emission increases. The central density limit is independent of the radius of the conducting cylinder; the required cathode emission for a given density $n(0)$ depends on the exterior potential drop of Eq. (5), but the interior solution is independently given by Eq. (3). Similarly, we note that even near the higher density edge ($\rho \simeq 2$), the density changes completely on the scale of a local Debye length

$$\frac{1}{n(r)} \frac{\partial n(r)}{\partial r} \lambda(r) = \frac{\rho}{2}.$$

Davidson[4] has demonstrated that a specific class of nonneutral rigid-rotor equilibria exhibit test charge shielding on the scale of a Debye length. Here, the small radial extent of the column would limit the "plasma" property of shielding. [It must be noted that this result is only true for an equipotential cathode. Reference 2 describes a nonneutral plasma with $R_p \gg \lambda(0)$ obtained from a cathode with a bias which is a function of radius.]

An interesting, and experimentally relevant, variation of the case we have considered would be to accelerate the cathode electrons into the tube, instead of relying on thermal velocities.[5] This could be accomplished by placing a grounded grid immediately in front of the cathode, and biasing the cathode negatively. The injected distribution of electrons is then an accelerated half-Maxwellian, each thermal electron being boosted equally in energy by the accelerating bias potential. For small injection currents, the space charge is negligible; whereas, for large enough injection current, the resulting space charge potential will reflect some of the injected beam. At any radius, a monotonically decreasing (in z) space charge potential would slow the beam until it became a half-Maxwellian distribution again, at precisely the cathode potential. A further decrease in potential would result in decreased density by reflecting the slowest particles, again giving $n \propto \exp(eV/kT)$. Since all particles are reflected by the negative anode, the full distribution is symmetric in $\pm v_z$. For large enough injected current, the (z-independent) potential at all radii would be sufficiently negative to slow the beam to thermal velocities, and the previously obtained radial solution would be applicable. An estimate of the required injection current density j for any acceleration voltage V_0 can be obtained by solving for $j = n(\rho_p)\bar{v}e$ and $\Phi(\rho_w) - \Phi(\rho_p) = eV_0/kT$, with $\bar{v}$ being the mean thermal velocity. For example, with the parameters $R_w = 2$ cm, $R_p = 1$ cm, $V_0 = 2$ V, $kT = \frac{1}{8}$ eV, an injection current of 51 μA/cm^2 would be required.

It should be noted that the possibility of unbounded potential and density would be limited by physical effects outside the scope of this analysis. These include limited cathode emission density and uniformity, finite B_0 effects,[6] and velocity-space microinstabilities.

The radial solutions presented here have assumed the existence (and stability) of a potential everywhere monotonic in z. We have investigated this assumption using a computer model of a *thin* axial tube of "thermal charge fluid" with self-consistent density and potential. Preliminary results are as follows. If the anode bias is not more negative than any interior potential, we see self-consistency problems which may be modeling space-charge relaxation oscillations.[7] For sufficiently negative anode, however, a stationary monotonic potential is found for injection of either a half-Maxwellian or an accelerated half-Maxwellian. These results suggest that the postulated potential configurations are physically realizable.

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