Fully Self-Consistent Calculation of Neoclassical Transport in Nonneutral Plasmas

T O'Neil, A. Kabantsev, D. Dubin
UC San Diego

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Neoclassical transport is enhanced flux across the magnetic field caused by symmetry-breaking "field errors" in the confinement fields

- In non-neutral plasmas, these neoclassical fluxes are often the dominant plasma loss process
- New results:

1. For known applied field errors that are not too big and do not produce localized particle trapping, experimentally observed radial particle transport is explained by the plateau regime
2. To achieve accurate predictions precise self-consistent potentials must be calculated numerically, including finite length effects in realistic geometry. This was done using a new code described below.
3. Dependence of the transport on plasma length and magnetic field has been characterized for two types of errors: a potential asymmetry applied to a wall electrode, and a tilt of the magnetic field compared to the axis of the Penning trap
4. For shorter plasmas in strong B fields, plateau transport is strongly suppressed by a novel effect: a minimum axial bounce frequency exists in short plasmas that can be larger than the rotation frequency if B is large, suppressing bounce-rotation resonances
5. This effect also explains why spheroidal plasma equilibria have been observed to have lower field error transport.

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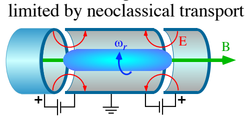
Nonneutral Plasmas have exceptional confinement properties, limited by neoclassical transport



A positive "pure ion plasma" is confined by static electric and magnetic fields of a Penning-Malmberg trap. Magnetic field is produced by a solenoid (not shown).

Axial confinement is due to electric fields from applied potentials on the end cylinders.

Radial confinement is due to the vxB force from plasma rotation. Typical rotation rates are $\frac{\omega_r}{2\pi} \sim$ 10-100 kHz

Confinement is limited by small field errors that break the cylindrical symmetry of the applied fields

These errors produce forces that torque on the plasma rotation ω_r slowing it and reducing the confining vxB force. This causes plasma expansion and eventual loss to the walls

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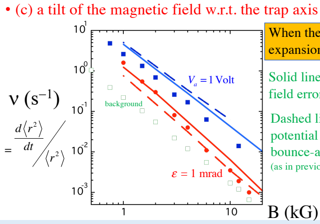
Experimental data (dots and squares) and 2 theories (solid & dashed lines) on radial expansion rate ν versus B in electron plasma from three types of field errors:

- (a) unknown "background errors" in the trap fields
- (b) voltage $V_a \cos 2\theta$ applied to a single wall electrode, length 4cm, $V_a = 1$ Volt
- (c) a tilt of the magnetic field w.r.t. the trap axis by angle ε= 1 mrad



When the error is known, we can predict the expansion rate within ~ 2

Solid lines: theory uses self-consistent field error potential calculated exactly

Dashed lines: self-consistent field error potential calculated keeping only the bounce-averaged plasma response (as in previous work*)

*Dubin,Kabantsev,Driscoll Phys. Plas. 19 (2012)

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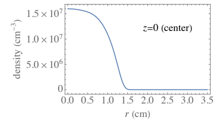
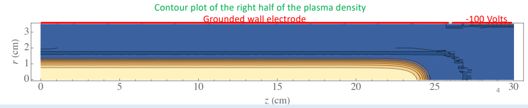
Ingredients in the theory:

1) Calculate realistic self-consistent plasma equilibrium for given measured radial density profile , assuming temperature T= 1 eV



Solve the nonlinear Poisson-Boltzmann system:
$n_0(r,z) = n_0(r,0)\exp[-e(\phi_0(r,z)-\phi_0(r,0))/T]$;
$\nabla^2\phi_0 = 4\pi e n_0(r,z)$
using an iterative relaxation method on an r,z grid

Contour plot of the right half of the plasma density



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2) Using this equilibrium density and potential, solve for the radial flux from a small applied electrostatic potential error on the wall, $\delta\phi(r$=3.5 cm, θ, z). 3 approaches were used:

a) "Brute force". Solve the linearized guiding center Fokker-Plank /Poisson system for the self-consistent perturbed distribution function $\delta f(r,z,v_z)$, assuming a uniform collision rate γ. The perturbed distribution function is assumed to satisfy

$$\delta f \equiv (g - e\delta\phi / T) n_0 f_{maxwellian}$$

where g is the non-adiabatic portion of the plasma response, which satifies

$$v_z \frac{\partial g}{\partial z} - \frac{\partial \phi_0}{\partial z}\frac{\partial g}{\partial p_z} + \omega_E \frac{\partial g}{\partial \theta} + C(g) = \frac{\omega_F}{T}\frac{\partial \delta\phi}{\partial \theta},$$

and where

$\omega_E = \frac{c}{Br}\frac{\partial \phi_0}{\partial r}$: ExB rotation rate, $\omega_F = \omega_E - \frac{cT}{eBrn_0}\frac{\partial n_0}{\partial r}$: fluid rotation rate (incl. diamagnetic drift),

and $C(g) = \gamma\left(\frac{\partial^2 g}{\partial v_z^2} - \frac{mv_z}{T}\frac{\partial g}{\partial v_z}\right)$: Fokker Planck collisions

$\delta\phi$ is determined self-consistently from δf: $\nabla^2 \delta\phi = -4\pi e \int dv_z \, \delta f$

radial flux is from θ-averaged radial ExB drift: $\Gamma_r = \frac{c}{2\pi Br}\int d\theta \, dv_z \, \delta f \frac{\partial \delta\phi}{\partial \theta}$

Solve this system iteratively on an r,z,v_z grid: choose a $\delta\phi$, evaluate δf, reevaluate $\delta\phi$ via a relaxation algorithm, repeat).
Then reduce the collision rate γ until resulting radial flux is independent of γ (plateau regime)

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b) Evaluate δf directly in the plateau regime using action-angle variables appropriate to the plasma equilibrium. The resulting flux should agree with method a) (it does!). The solid lines in the radial expansion rate figures use both method a and b; the difference between the methods is negligible.

Here we drop collisions and write the perturbed Vlasov equation in action angle variables (ψ,I). The equation can then be solved analytically for a given perturbed potential:

$$g = \sum_{n,l} e^{in\psi + il\theta} g_{n,l}(r,I), \quad \delta\phi(r,z,\theta) = \sum_{n,l} e^{in\psi + il\theta}\delta\phi_{n,l}(r,I)$$

$$\Rightarrow il\bar{\omega}_E(r,I) g_{n,l} + in\omega_b(r,I) g_{n,l} = il\frac{\omega_F}{T}\delta\phi_{n,l}$$

$$\Rightarrow g_{n,l} = \frac{\omega_F}{T}\frac{l\delta\phi_{n,l}}{l\bar{\omega}_E + n\omega_b} \quad (*)$$

$\omega_b(r,I)$: axial bounce frequency
$\bar{\omega}_E(r,I)$: bounce – averaged ExB rotation frequency

Take as the perturbed potential the output of the code from "brute force" method a). We do not try to iteratively solve for a self-consistent g and $\delta\phi$ using the above expression. Divergences in g at bounce-rotation resonances are difficult to deal with, as are $(z, v_z) \leftrightarrow (\psi, I)$ transformations at each radial grid point. (The Plemelj formula can still be used to obtain the radial flux from the above expression for g.)

c) Redo method b) by determining $\delta\phi$ *in a more approximate manner:* Approximate δf by it's bounce-averaged form in the collisionless Vlasov equation (the n=0 term in (*)), which removes all bounce-rotation resonances. Use this to calculate an approximate self-consistent perturbed potential $\delta\phi$ iteratively. Then use this approximate bounce-averaged form for $\delta\phi$ in plateau regime calculation b) or in a). This approximate method yields the dashed lines in the previous expansion rate figure.

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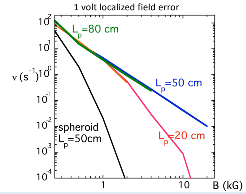
Radial Transport at different plasma lengths+for a spheroid

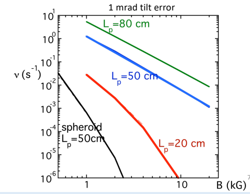
We varied the half-length L of the electrode structure: L=15 cm, L=30 cm(previous data) and L=45 cm, keeping the end electrodes at 100Volts and the radial density profile roughly the same. This makes cylindrical plasmas with three (full) lengths: Lp~ 20 cm, Lp~50 cm (previous), and Lp~ 80 cm.

We also created a 50 cm long spheroid using a tailored wall voltage, with roughly the same radial density profile

1 volt localized field error



1 mrad tilt error



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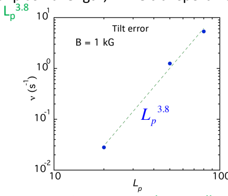
- At or below B~1 kG, transport from a localized error is roughly independent of plasma length, while transport from a tilt error scales roughly as $\nu \sim L_p^{3.8}$



- Above B~1 kG, transport is suppressed at smaller L_p. For a spheroid it is suppressed even more.

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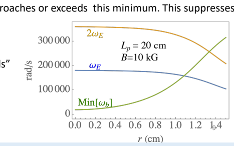
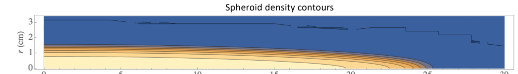
Suppression of radial transport at smaller L_p and larger B

- Radial transport is dominated by flux at larger radii within the plasma.
- At these larger radii, and for shorter plasmas, the axial equilibrium electric field is not fully Debye shielded, making a weak harmonic axial potential well near z=0 (plasma axial center).
- This causes a minimum value of the axial bounce frequency at low particle kinetic energies
- For sufficiently large B, the plasma rotation frequency approaches or exceeds this minimum. This suppresses bounce-rotation resonances where $n\omega_b(r,I) = l\bar{\omega}_E(r,I)$
- This finite length effect is missed in previous work that uses periodic boundary conditions in z or "flat ends"

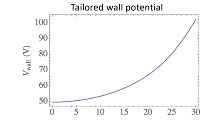


Suppression effect is even greater in spheroids

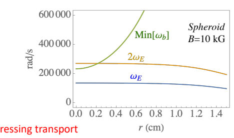
Spheroid density contours



Tailored wall potential



This wall potential creates much larger axial harmonic well inside plasma making minimum bounce frequency larger:



A magnetic well might be even more effective at suppressing transport since magnetic nonuniformity is not Debye shielded –future work

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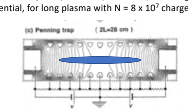
One experiment* also observed a reduced expansion rate when the plasma is confined in a harmonic potential in z (including plasma image charge effects)

Multi-ring Penning trap with ring voltages chosen to produce $\phi = C(z^2 - r^2/2)$ near axis, including image charge potential, for long plasma with N = 8 x 10^7 charges



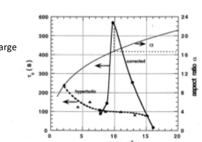


Fig. 6. Confinement times τ_s for different N's for the case with the correction (solid circles) and also the case without the correction (triangle plots). Here, the correction was made for $N = 8 \times 10^7$. The solid curve is α in the corrected case.

*Higaki et al., Jpn J. Appl. Phys. 37 664 (1997)