ION-INDUCED INSTABILITY OF DIOCOTRON MODES IN ELECTRON PLASMAS MODELLING CURVATURE-DRIVEN FLUTE MODES

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We study ion-induced instability of flute-like ($k_z \approx 0$) diocotron modes in pure electron plasmas confined in a cylindrical Penning-Malmberg trap. In the absence of positive ion contamination, the low m_θ diocotron modes are either neutrally stable (for $m_\theta = 1$) or weakly damped (for $m_\theta = 2,3\ldots$) by Landau resonance on electrons co-rotating with the diocotron waves. By adding a small fraction (< 1%) of positive ions into a double-well confinement configuration, we observe exponential instability of low m_θ diocotron modes. The growth rates γ_m are directly proportional to the overall ion fraction, N_i/N_e, and proportional to an effective charge separation of electrons and ions in the periodic wave perturbation.

I. INTRODUCTION

Curvature-driven flute modes have been a problem for magnetic confinement from the beginning of fusion research[1]. The charge separation produced by a curving magnetic field causes rapid loss of particles from **E**×**B** drifts. Magnetically confined *pure* electron plasmas are naturally free from this problem, until a small fraction of positive ions is added. Then, the ions trapped in a double-well confinement configuration may destabilize the electron plasma diocotron modes, analogous to the curvature driven destabilization of neutral plasma flute modes.

It is of general interest to see the onset of the flute (interchange) instability in the transition between these two ($N_i = 0$ and $N_i \approx N_e$) extremely different regimes. Also, this strong instability may have important implications for a variety of experiments, such as the double-well traps to overlap the positron and anti-proton clouds for production of anti-hydrogen[2,3].

In our experiments we use a Penning-Malmberg trap[4] in a double-well (nested) configuration (see Fig. 1). The electron column of length $L_e \leq 50$ cm is contained inside a stack of hollow conducting cylinders of radius $R_w = 3.5$ cm, which reside in an ultrahigh vacuum with residual pressure ~ 10^{-11} Torr. The inner end cylinders are negatively biased (−100 V) with respect to the electron plasma potential ($\phi_p \approx -30$ V) to axially confine electrons, while the outer end cylinders are grounded or biased positively (+40 V) to axially confine ions. A strong axial magnetic field ($B \leq 20$ kG) ensures radial confinement both for the electrons and ions.

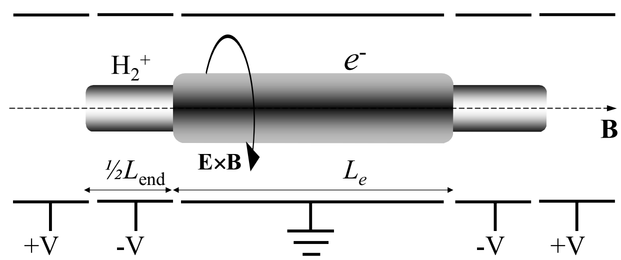

Fig. 1. Schematic of the double-well (nested) trap.

Typical electron columns have central density $n_e \approx 1.5 \cdot 10^7$ cm^{-3} over a bell-shaped radial profile with a characteristic radius $R_p \approx 1.2$ cm, giving line density $N_e \equiv n_e \pi R_p^2 \approx 7 \cdot 10^7$ cm^{-1}. The unneutralized electron charge results in an **E**×**B** rotation of the column at frequency $f_R = cE/2\pi rB$ ~ (0.1 MHz)(2kG/B). These pure electron plasmas have exceptional confinement properties and can be contained for hours[5].

The positive ion population (H_2^+) is created either by controlled external injection of an ion beam into cold ($T_e \approx 0.5$-1 eV) electron columns; or by ionization of the background gas within rf-heated ($T_e \approx 7$-9 eV) electron plasmas.

The flute instability in neutral plasmas is driven by the drift charge separation due to curving **B**-field; here the instability is driven predominantly by the bounce-averaged **E**×**B** drift charge separation due to the difference in confinement regions for the electrons and ions in the double-well trap. Let us to consider a cartoon of this process based on as yet unjustified assumption that the ion motion becomes stochastic on the diocotron time scale $\tau_d = 1/f_m$. In Figure 2, for visual simplicity we consider the $m_\theta = 1$ diocotron perturbation (off-axis shift) of amplitude d. However, the very same basic reasoning is valid for any m_θ diocotron modes. In the shown double-well configuration the trapped ions spend a part τ_{end} of their overall bounce time τ_{bnc} outside the trapped electron column, where they lag in phase behind the electron diocotron density perturbation by a typical phase shift $\theta_d = 2\pi\tau_{end}/\tau_{bnc}$, which is accumulated over a full diocotron period.

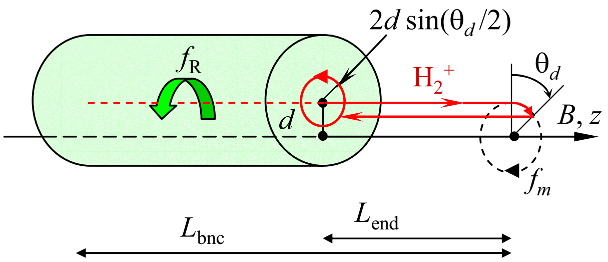

Fig. 2. Cartoon of the phase shift θ_d and the corresponding displacement $2d\sin(\theta_d/2)$ acquired by an average ion during diocotron cycle in the double-well trap.

Note that symmetry dictates that the end drift of the whole ion fraction is well represented by the drift of an ion coming from the electron column center. Upon their return into electron column, after being smashed by the plasma rotation along a circular orbit of radius $2d\sin(\theta_d/2)$, the ions acquire a change in their mean-square-radius (MSR) equal to $\delta r^2 = 2d^2(1\text{-}\cos\theta_d)$. Since we have assumed that the ion motion becomes stochastic on the diocotron time scale $\tau_d = 1/f_m$, the ions diffuse in the oscillating m_θ-wave perturbation at a rate $D_m \equiv \delta r^2 f_m$, which results into the net change of the MSR accumulated by the whole ion fraction N_i during their active lifetime τ_{lf} in the electron column equal to

$$N_i\Delta r^2 \equiv N_i D_m \tau_{lf} = N_i f_m \tau_{lf} \times \left(1-\cos\theta_d\right). \quad (1)$$

According to the conservation of canonical angular momentum[6], the rate of change in the MSR of ions has to be balanced by the rate of change in the MSR of the whole electron column, which moves further off-axis as

$$N_i D_m = N_e 2\gamma_m d^2. \quad (2)$$

This gives us the instability with the m_θ-mode growth rate in the form

$$\gamma_m = \left(N_i/N_e\right) f_m \times \left[1-\cos\left(2\pi\tau_{end}/\tau_{bnc}\right)\right]. \quad (3)$$

Using partial neutralization equality for the diocotron modes, $\Delta f_m/f_m = N_i/N_e$, we can rewrite γ_m as

$$\gamma_m = \Delta f_m \times \left[1-\cos\left(2\pi\tau_{end}/\tau_{bnc}\right)\right], \quad (4)$$

which is much more convenient for comparison to the experiment, since both γ_m and Δf_m can be simultaneously measured. This result is rather remarkable in its simplicity: the growth rate of any diocotron mode is defined by its frequency, by plasma's fractional neutralization, and by the bounce-averaged phase shift between the ion and electron **E**×**B** drifts.

In the straightforward comparison to characteristic growth rate of classical flute modes[1], $\gamma \sim \upsilon_T/L_B$ (where L_B is a scale length for the axial B-field variations), the double-well traps case is equivalent to an effective $L_B \sim R_p/[1\text{-}\cos(2\pi\tau_{end}/\tau_{bnc})]$, which may get extremely short ($L_B \sim R_p/2$) from the mirror traps point of view.

Note that the growth rate γ_m for the trapped-ion-induced instability of diocotron modes is also functionally similar to the growth rate of the passing-ion-induced instability of diocotron modes[7], which may be expressed as $\gamma_{sp} = (N_i/N_e) f_{sp}\times[1\text{-}\cos(2\pi f_R/f_{sp})]$. Here, $f_{sp} \approx 1/\tau_{bnc}$ is the ion single pass time through the electron column, and f_R is the electron column rotation frequency. This passing-ion-induced instability gives us the background growth rate in case of modulated ion trapping experiments.

II. SOME EXPERMENTAL RESULTS

The growth rates $\gamma_m(t)$ for the $k_z = 0$, $m_\theta = 1,2,3\ldots$ diocotron modes are measured by digitizing the amplitude $A_m(t)$ of corresponding wall potentials induced by the diocotron oscillations at the azimuthally sectored electrodes. These amplitudes are verified (and calibrated) by taking corresponding (r,θ) moments of 2D-density distribution of the dumped electron column from a CCD camera diagnostic[8]. In the experiments, we usually keep the mode amplitudes, $d_m(t) \equiv A_m(t)/R_w$ (scaled by the wall radius R_w), small enough ($d_m \le 0.01$) to have nonlinear effects well constrained. The trapped ion fraction $N_i(t)/N_e$ is obtained simultaneously with $\gamma_m(t)$ by measuring the relative frequency change of the $m_\theta = 1$ diocotron mode $\Delta f_1(t)/f_1$ during modulated ion beam trapping, since fractional neutralization gives frequency change as $\Delta f_1 = (N_i/N_e) f_1$.

Figure 3 demonstrates the 2-decade exponential growth of the $m_\theta = 1$ diocotron mode in linear regime. Here, the ion fraction $N_i/N_e \approx 5\cdot 10^{-4}$, and $f_1 \approx 4$ kHz ($B = 7$ kG). Note that $f_1 \approx f_R(R_p/R_w)^2$. Modes with higher m_θ-wave numbers show faster initial growth in accord with their higher frequencies[9] $f_m \approx [m_\theta\text{-}1 + (R_p/R_w)^2] f_R$, i.e., $\gamma_m/\gamma_1 \approx f_m/f_1$, but then they rapidly approach nonlinear saturation due to spatial Landau damping in the radial edge of the plasma column.

While frequencies of diocotron mode are obvious function of B ($f_m \propto 1/B$), their growth rates, normalized by the ion production (injection current) rate per electron, ν_i, show no dependence on B (see Fig. 4). This is consistent with the measured linear B-scaling of the active ion lifetime $\tau_{lf} \equiv N_i/N_e\,\nu_i$ in electron column (Fig. 5), which also indicates an **E**×**B** drift nature of the ion radial losses. For our plasma and trap parameters it takes from hundreds to thousands of diocotron cycles for an average ion to reach the radial edge of electron plasma, and details of this process are still ill understood.

Besides the rather trivial linear dependence of γ_m on the ion fraction, we have also qualitatively verified the $\sin^2$-dependence, $\gamma_1/\Delta f_1 = 2\sin^2(\pi\tau_{end}/\tau_{bnc})$, on the effective charge separation. The most simple double-well configuration (adjacent cylinders used to confine electrons and ions) typically gives us $\tau_{end}/\tau_{bnc} \approx 0.1$, which should result into the slope $\gamma_1/\Delta f_1 \approx 0.2$. Figure 6 shows this is quite reasonable estimate. Increasing τ_{end} by adding cylinders to the ion part of the trap and simultaneously shrinking the length of the electron column, we first have observed a gradual increase of the $\gamma_1/\Delta f_1$ slope up to 1.6 ($L_e \approx L_{end}$), which then been followed by a decrease down again to about 0.2 at $\tau_{end}/\tau_{bnc} \approx 0.9$. More quantitative comparison is complicated by the coarseness of confinement length steps and the ill-defined dependence τ_{end} on the ion energy and axial potential distribution.

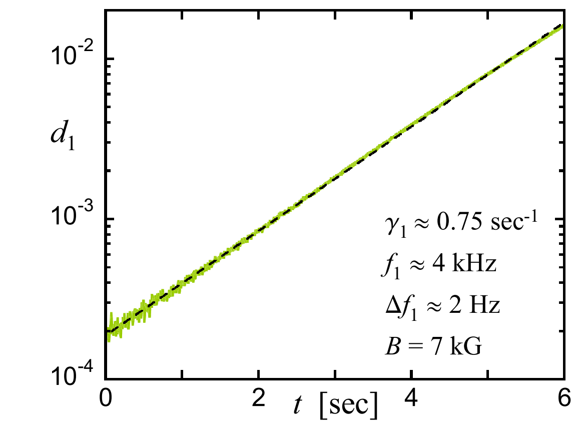

Fig. 3. Exponential growth of the $m_\theta = 1$ diocotron mode in linear regime ($d_1 \leq d_{cr} << 1$).

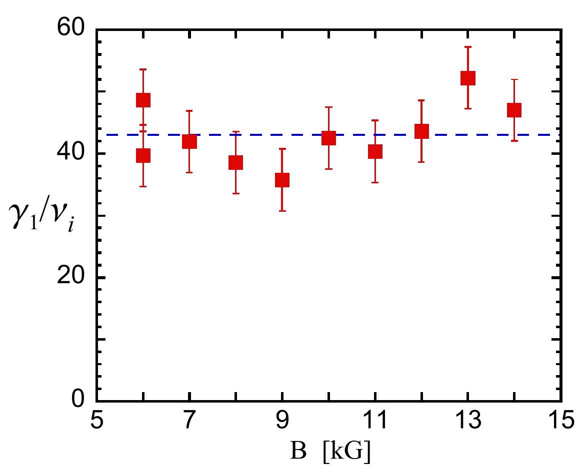

Fig. 4. Growth rate per injected ion is independent of the magnetic field. Big error bars are due to large uncertainty in measurements of the ion injection rate at pA level.

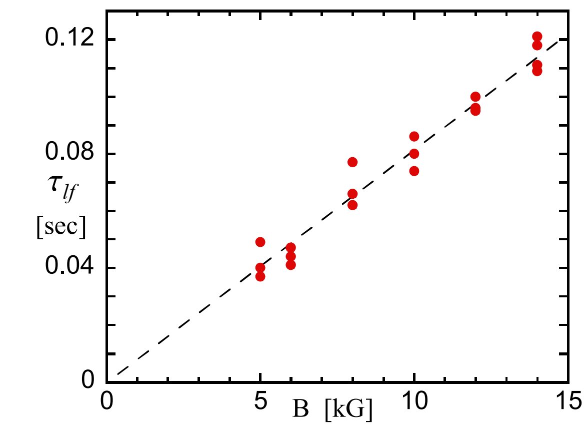

Fig. 5. Active lifetime τ_{lf} of an average ion as a linear function of the magnetic field. The lifetime is measured from exponential saturation of fractional neutralization $\Delta f_2(t)$ after the beginning of ion injection, $\Delta f_2(t) = -f_2(0)\nu_i \tau_{lf}[1 - \exp(-t/\tau_{lf})]$.

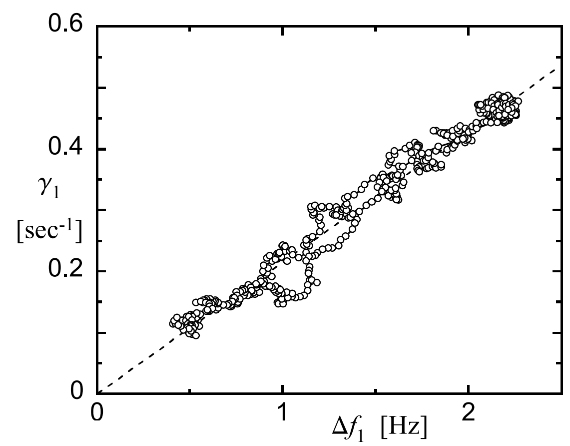

Fig. 6. Growth rate as a linear function of fractional neutralization ($\gamma_1/\Delta f_1 = 0.22$, $f_1 = 3889$ Hz at $N_i = 0$).

III. CONCLUSIONS

Instabilities of diocotron modes are commonly observed when small ion fractions are added to pure electron plasmas[10-16]. However, historically[10] these instabilities were first thought to be driven by the difference in rotation frequencies of the two species caused by inertial (mass difference) effects. This is analogous to the centrifugal MHD-instabilities in neutral plasmas, which are usually less dangerous than magnetic curvature driven (charge difference) modes if they do not belong to a narrow parameter range (resonant effects).

Here, we find that the ion-induced instability is driven by an entirely different physical mechanism.

Broadly, the different bounce-averaged azimuthal drifts of electrons and ions tends to polarize the diocotron mode density perturbations, thereby developing instability similar to the classical flute MHD-instability of neutral plasmas confined in non-uniform magnetic fields. The major factor that distinguishes charged plasma behavior in our experiment from neutral plasma confinement is that this charge separation mainly comes from the two species sampling different radial electric fields at the plasma column ends (prevalent importance of this effect for double-well traps was first mentioned by Pasquini and Fajans[15,16]). However, it allows us an easy control over the effective drift separation of oppositely charge particles in the wave perturbation, by simply varying the ratio of the ion end transit time τ_{end} (by adjusting the end length L_{end} or the potentials) to the total bounce time τ_{bnc}.

As a result, the measured trapped-ion-induced growth rates show a rather simple dependence

$$\gamma_m = \left(N_i/N_e\right) f_m \times \left[1-\cos\left(2\pi\tau_{end}/\tau_{bnc}\right)\right]$$

verified in a broad range of relevant plasma and trap parameters. Having just the linear reduction factor, N_i/N_e, this strong exponential instability has no threshold on the smallness of the ion fraction, and without paying proper attention it may lead to significant rate of particle radial transport and losses in double-well confinement configurations.

ACKNOWLEDGMENTS

This work was supported by National Science Foundation Grant No. PHY0354979.

REFERENCES

[1] M. N. ROSENBLUTH and C. L. LONGMIRE, "Stability of Plasmas Confined by Magnetic Fields," *Annals of Physics*, **1**, 120 (1957).

[2] G. GABRIELSE *et al*., "Background-Free Observation of Cold Antihydrogen with Field-Ionization Analysis of Its States," *Phys. Rev. Letters*, **89**, 213401 (2002).

[3] M. AMORETTI *et al*., "Complete Nondestructive Diagnostic of Nonneutral Plasmas Based on the Detection of Electrostatic Modes," *Phys. Plasmas*, **10**, 3056 (2003).

[4] A. A. Kabantsev *et al*., "Trapped Particles and Asymmetry-Induced Transport," *Phys. Plasmas*, **10**, 1628 (2003).

[5] C. F. DRISCOLL, K. S. FINE and J. H. MALMBERG, "Reduction of Radial Losses in a Pure Electron Plasma," *Phys. Fluids*, **29**, 2015 (1986).

[6] T. M. O'NEIL, "A Confinement Theory for Nonneutral Plasmas," *Phys. Fluids*, **23**, 2216 (1980).

[7] A. A. KABANTSEV and C. F. DRISCOLL, "Fast Measurements of PicoAmp Plasma Flows Using Trapped Electron Clouds," *Rev. Sci. Instrum.*, **75**, 3628 (2004).

[8] K. S. FINE, W. G. FLYNN, A. C. CASS, and C. F. DRISCOLL, "Relaxation of 2D Turbulence to Vortex Crystals," *Phys. Rev. Letters*, **75**, 3277 (1995).

[9] S. A. PRASAD and T. M. O'NEIL, "Waves in a Cold Pure Electron Plasma of Finite Length," *Phys. Fluids*, **26**, 665 (1983).

[10] R. H. LEVY, J. D. DAUGHERTY, and O. BUNEMANM, "Ion Resonance Instability in Grossly Nonneutral Plasmas," *Phys. Fluids*, **12**, 2616 (1969).

[11] A. J. PEURRUNG, J. NOTTE, and J. FAJANS, "Observation of the Ion Resonance Instability," *Phys. Rev. Letters*, **70**, 295 (1993).

[12] J. FAJANS, "Transient Ion Resonance Instability," *Phys. Fluids B*, **5**, 3127 (1993).

[13] M. R. STONEKING, M. A. GROWDON, M. L. MILNE, and R. T. PETERSON, "Poloidal ExB Drift Used as an Effective Rotational Transform to Achieve Long Confinement Times in a Toroidal Electron Plasma," *Phys. Rev, Letters*, **92**, 095003 (2004).

[14] G. BETTEGA *et al*., "Experimental Investigation of the Ion Resonance Instability in a Trapped Electron Plasma," *Plasma Phys. Contr. Fusion*, **47**, 1697 (2005).

[15] T. PASQUINI and J. FAJANS, "Ion Resonance Instability in a Double Well Trap," *Programme & Abstracts of 3rd International Conference on Trapped Charged Particles and Fundamental Interactions*, Wildbad Kreuth, Germany, August 2002, p. 83, 292nd WE-Heraeus Seminar (2002).

[16] T. PASQUINI and J. FAJANS, "Ion Resonance Instability in a Double Well Trap," *Bull. Am. Phys. Society*, Orlando, Florida, November 2002, Vol. 47, No. 9, p. 127, AIP (2002).