Effect of squeeze on electrostatic TG wave damping

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Abstract. We present a 1D theory, neglecting radial dependency, for the damping of cylindrically symmetric plasma modes due to a cylindrically symmetric squeeze potential $V_{sq}(z)$, applied to the axial midpoint of a non-neutral plasma column. Inside the plasma, particles experience a much smaller, Debye shielded squeeze potential $\varphi_0(z)$ of magnitude φ_s. Squeeze divides the plasma into passing and trapped particles; the latter cannot pass over the squeeze. Both analytical and computer simulation methods were used to study a 1D squeezed plasma mode. For our analytical study, in the regime where $q\varphi_s/T \ll 1$, we assume the trapped particle population to be negligibly small and we treat $q\varphi_0(z)$ as a pertubation in the equilibrium hamiltonian. Our computer simulations consist of solving the 1D Vlasov-Poisson system and obtaining the damping rate for a self-consistent plasma mode. Damping of the mode in collisionless theory is caused by Landau resonances at energies E_n for which the bounce frequency $\omega_b(E_n)$ and the wave frequency ω satisfy $\omega = n\omega_b(E_n)$. Particles experience a non-sinusoidal wave potential along their bounce orbits due to the squeeze potential. As a result, squeeze induces bounce harmonics with $n \gg 1$ in the perturbed distribution. The harmonics allow resonances at energies $E_n \leq T$ and cause a substantial damping, even at wave phase velocities much larger than the thermal velocity, which is not expected for unsqueezed plasma. In the regime $\omega/k \gg \sqrt{T/m}$ (k is the wave number) and $T \gg q\varphi_s$, the resonance damping rate has a $|V_{sq}|^2$ dependence. This behavior is consistent with the observed experimental results.

Keywords: Trivelpiece-Gould modes, Landau damping, nonneutral plasma
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INTRODUCTION

Trivelpiece-Gould modes are plasma modes which are modified by the cylindrical confinement geometry of the trap. Experiments have been performed on the effect of an externally applied cylindrically symmetric electrostic potential(squeeze potential), on the damping of $l = 0$(azimuthally symmetric) $m = 1$ Trivelpiece-Gould modes, in a thin and long finite length plasma. In the absence of squeeze, for a plasma of radius r_p and length L, contained in a perfectly conducting cylinder of radius r_w, we have the following dispersion relation up to the lowest order thermal correction [1]:

$$\omega \approx \omega_p \frac{k_m}{k_\perp}\left[1+\frac{3}{2}\left(\frac{v_T}{v_{ph}}\right)^2\right] \tag{1}$$

The above equation employs the following notation: mode frequency ω, plasma frequency $\omega_p = \sqrt{4\pi q^2 n_0/m_q}$, density n_0, particle charge q, particle mass m_q, phase velocity $v_{ph} = \omega/k_m$, thermal velocity $v_T = \sqrt{T/m_q}$, radial wave number $k_\perp \approx (r_p)^{-1}\sqrt{\frac{2}{\log(r_w/r_p)}}$ and axial wave number $k_m = \pi m/L$.

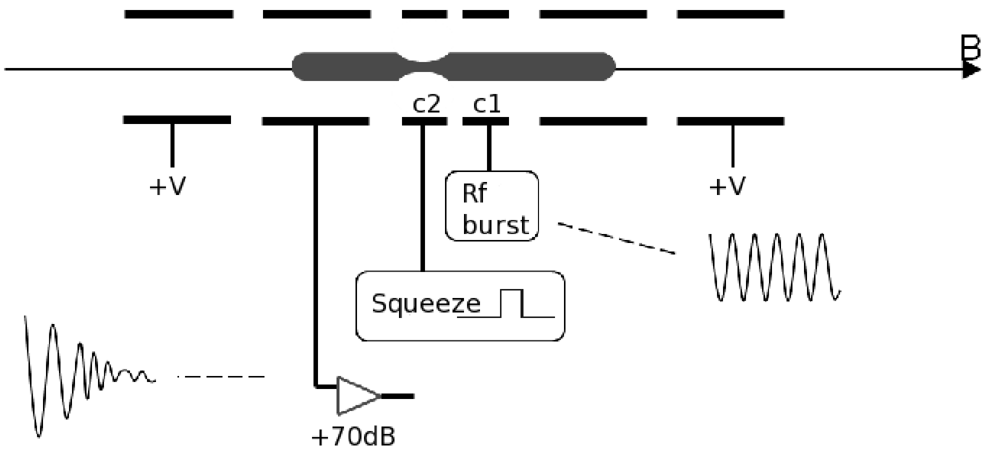

FIGURE 1. Schematic depiction of the experimental setup.

We present a 1D model theory for squeezed Trivelpiece-Gould modes, which includes a self-consistent treatment of mode potential in the presence of a 1D squeeze potential acting as a kinetic barrier for the bounce motion of particles along the plasma axis. Figure (1) demonstrates the confinement geometry used in the experiments. The plasma is confined in a Malmberg-Penning trap consisting of a conducting cylinder, axially divided into a number of sections. Axial confinement is provided by applying an electrostatic potential $+V$(for ions) to the end sections. Radial confinement is provided by a strong uniform magnetic field B directed along the axis of cylinder. The plasma resides in the inner conducting sections, which are grounded.

The middle section of the trap is divided into two parts, c1 and c2. Trivelpiece-Gould modes are excited by applying a sinusoidal voltage to cylindrical sector c1. On a separate electrode which works as an antenna, voltage is measured. This voltage is induced by the wave density perturbation δn. Damping of the excited wave is directly measured from the decay rate of the receiving signal. Damping of a small amplitude wave is expected to be exponential in time. Next, a squeeze potential is applied to cylindrical sector c2, for a time interval of ≈ 10 ms, during which damping of the wave is substantially enhanced. Turning off the squeeze potential lowers the damping rate to a small value, which is however greater than the value it had preceeding the application of squeeze. This can be explained by the heating of plasma as the wave is damped. Experiments were performed on TG modes with phase velocities much greater than the thermal velocity. In such conditions, Landau damping of the mode in the absence of squeeze is expected to be negligibly small. However, the damping rate of the TG modes shows a square dependence on the magnitude of the applied squeeze. This behavior was observed over a relatively large temperature range($T = 10\text{K} - 1000\text{K}$). In hotter plasmas the damping rate of the mode due to squeeze was more enhanced compared to the colder plasmas.

MODEL

We make a few appoximations and simplifications to come up with a tractable theoretical model. We assume that the squeeze potential is symmetric in z with respect to the center of plasma. This is not the case in the experiments, however this added symmetry simplifies the problem. Density and potential inside the equilibrium plasma are functions of r and z. We neglect radial variation for simplicity and we assume plasma ends are flat and that particles undergo specular reflection at the ends, at $z = \pm L/2$. Similar to the experiments, dimensions of plasma and trap are ordered as $r_p \leq r_w \ll L$. In this range of parameters we can see from (1) that $\omega \ll \omega_p$. Furthermore, time scale ordering is chosen in accordance with the experiments. For the case of azimuthally symmetric modes in a strong magnetic field we have the following ordering for the time scales:

$$\nu_{col} \ll \omega \ll \omega_p \ll \omega_c \tag{2}$$

Here, ν_{col} is the collision frequency and $\omega_c = qB/m_q c$ is the cyclotron frequency. In a strong magnetic field the distribution of particle guiding centers is described by drift-kinetic equations. Particle motion consists of $E \times B$ drift motion across magnetic field and streaming along the magnetic field in z direction. For a cylindrically symmetric mode the time evolution of linear pertubations is given by 1D Vlasov equation:

$$\partial_t \delta \mathrm{f} + v \partial_z \delta \mathrm{f} - \frac{q}{m_q} \partial_z \varphi_0 \partial_v \delta \mathrm{f} - \frac{q}{m_q} \partial_z \delta\varphi \partial_v \mathrm{F}_0 = 0 \tag{3}$$

where v is velocity in axial(z) direction, F_0 is the equilibrium distribution function, φ_0 is the equilibrium potential, and $\delta\varphi$ is the mode potential perturbation.

Equilibrium density and potential of a squeezed plasma are functions of both r and z, and as a result mode potential is also a function of both of these coordinates. Effect of radial confinement on the finite length plasma modes was studied by Prasad and O'Neil [2], in which linear modes were calculated as a first order perturbation in r_p/L and it was shown that zero'th order modes will become coupled due to finite radius effect and there will be corrections to damping rate of the mode. Adding squeeze to the problem has similar effects on the eigenmode and its damping rate. In order to focus on the effect of squeeze on the modes, as a simpler more tractable model, we consider a 1D plasma(i.e zero'th order in r_p/L) with a z dependent squeeze potential given by $V_{sq}(z)$. Nevertheless, we keep a radial wave number $k_\perp$ as a system parameter in order to maintain some effect of radial confinement on plasma modes(Debye sheilding due to $k_\perp$). The Hamiltonian of the equilibrium state of plasma is given by

$$H_0 = m_q v^2/2 + q\varphi_0(z) \tag{4}$$

$\varphi_0(z)$ is the Debye shielded squeeze potential inside the plasma with maximum value φ_s at the center of plasma. Hence, $\varphi_0(z)$ is the sum of the externally applied squeeze potential $V_{sq}(z)$ and the response of the plasma to the external potential $\varphi_{pe}(z)$.

$$\varphi_0(z) = V_{sq}(z) + \varphi_{pe}(z) \tag{5}$$

Passing particles have energies $E > \varphi_s$ and travel the whole length of the plasma in their bounce motion. These particles slow down when they climb up the kinetic barrier $q\varphi_0(z)$

and speed back up as they go down the kinetic barrier. Particles with energies $E < \varphi_s$ are trapped on left or right side of $q\varphi_0(z)$ and cannot cross over to the other side. We are interested in the regime where $q\varphi_s \ll T$. In this regime the trapped particle population is very small compared to the passing particle population and the effects we are concerned with are mainly due to passing particles. Therefore we neglect the trapped particles. The equilibrium distribution $\mathrm{F}_0(z,v)$ is given by the Boltzmann distribution:

$$\mathrm{F}_0(H_0) = \frac{e^{-\frac{H_0}{T}}}{\sqrt{2\pi T/m_q}\int_{-L/2}^{L/2} e^{-\frac{\varphi_0(z)}{T}} d(z/L)} \tag{6}$$

F_0 and φ_{pe} together satisfy Poisson-Boltzmann equation. We are particularly interested in a case where the squeeze potential is Debye shielded to the extent that the equilibrium potential energy inside plasma is much smaller than average kinetic energy i.e. $\varepsilon = q\varphi_s/T \ll 1$. In this situation, we can expand to first order in ε and get the following relation:

$$(-k_\perp^2 + \partial_z^2)\varphi_{pe} = \lambda_D^{-2}\left(\varphi_0 - \langle\varphi_0\rangle\right) \tag{7}$$

where $\lambda_D = \sqrt{T/4\pi q^2 n_0}$ is the Debye length. Hence, from (7) and (5) we can solve for φ_0 in terms of V_{sq}. As a result we can see that the magnitude of the potential inside plasma is linearly proportional to the magnitude of squeeze potential:

$$\varphi_s \propto |V_{sq}|, \quad q\varphi_s \ll T \tag{8}$$

For a long thin plasma where we have $\omega \ll \omega_p$, to the zero'th order in ω/ω_p, mode potential is flat at the ends of plasma i.e. $\partial_z\delta\varphi(\pm L/2) \approx 0$ [3]. Therefore, mode potential can be written as:

$$\delta\varphi(z,t) = \sum_{m=1}^{\infty} e^{-i\omega t}\delta\bar{\phi}_m \cos[k_m(z+L/2)] + c.c., \quad k_m = m\pi/L \tag{9}$$

Particles perform a periodic bounce motion along their unperturbed orbits and their canonical action variable is a constant of motion. Thus, in order to simplify our calculations we use canonical action-angle variables ψ and I and for the mode potential we can write:.

$$\delta\varphi(\psi,I;t) \;=\; \sum_{n=-\infty}^{\infty}\sum_{m=1}^{\infty} \delta\bar{\phi}_m C_m^n(I) e^{i(n\psi-\omega t)} + c.c. \tag{10}$$

where $C_m^n(I)$ is given by:

$$C_m^n(I) = \frac{1}{2\pi}\int_0^{2\pi} e^{-in\psi}\cos[k_m(z(\psi,I)+L/2)]d\psi \tag{11}$$

The mode potential $\delta\varphi$ can be obtained by simultaneously solving 1D Vlasov eq. (3) and Poisson equation:

$$(-k_\perp^2 + \partial_z^2)\delta\varphi = -4\pi q n_0 \int_{-\infty}^{\infty} \delta\mathrm{f}\; dv_z \tag{12}$$

where the mode perturbation to distribution function is of the form:

$$\delta \mathrm{f}(z,v;t) = \delta f(z,v)e^{-i\omega t} + c.c. \tag{13}$$

After performing some algebraic steps on Poisson equation (12) and Vlasov equation (3), we obtain the dispersion relation which can be written in a complex matrix eigenvalue equation form:

$$\mathbf{M}(\omega).\mathbf{a} = 0 \tag{14}$$

where dispersion matrix $\mathbf{M}$ and eigenvector $\mathbf{a}$ (whose elements are Fourier components in position space), are given by:

$$\mathbf{M}^{m,p}(\omega) = \delta_{m,p} + \chi_m^{-2}\sum_{n=1}^{\infty}\hat{\Pi}_n(\omega)C_p^n(I)C_m^n(I) \tag{15}$$

$$\mathbf{a}^T = (\delta\bar{\phi}_1, \delta\bar{\phi}_2, \dots) \tag{16}$$

where $\chi_n^2 = k_\perp^2 + k_n^2$ and we define the following short-hand notation:

$$\hat{\Pi}_n(\omega)g(I) = -\frac{4\pi\omega_p^2}{LT}\int_{\mho} dI\left(\frac{\omega_b \mathrm{F}_0(E)}{\omega/n - \omega_b} - \frac{\omega_b \mathrm{F}_0(E)}{\omega/n + \omega_b}\right)g(I) \tag{17}$$

where energy $E = E(I)$ through action-angle transformation. Eigenvalues satisfying (15) are the complex mode frequencies and eigenvectors are the Fourier components of $\delta\varphi(z)$. To deal with singularities, all integrals over action variable are to be performed along Landau contours, i.e. contours drop bellow the poles(as shown by the symbol $\mho$ in the above integrals). Different Fourier components of the mode(elements of RHS of (16)) are coupled through the non-diagonal elements of $\mathbf{M}(\omega)$. Details of this calculation will be presented in a separate publication.

Treating a small $q\varphi_0(z)$ compared to T as a perturbation

In order to obtain the eigenvalues, eigenvectors and damping rate of the modes, we take a peturbative approach. Assuming the parameter $\varepsilon = q\varphi_s/T$ to be small, the majority of particles have energies greater than $q\varphi_s$ and see the potential barrier as a small bump that gently slows them them down as they move along their bounce orbits. Thus, for $q\varphi_s \ll E$, from perturbation theory we can calculate $C_m^n(I)$'s which have the form:

$$C_m^n(I) = \frac{1}{2}\delta_{|n|,m} + \alpha_m^n(E) \tag{18}$$

$\alpha_m^n(E) \propto q\varphi_s/E$ is the first order in ε correction due to squeeze effect to the Fourier components in action-angle space for an orbit with energy $E \gg q\varphi_s$ where $E \approx \pi^2 I^2/2m_q L^2 + \langle\varphi_0\rangle$, and bounce frequency $\omega_b = \partial_I E = \pi^2 I/m_q L^2$. We have the following series expansions in terms of ε as small parameter:

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{M}_1 + \mathbf{M}_2 + \dots \quad (19)$$

$$\mathbf{a} = \mathbf{a}_0 + \mathbf{a}_1 + \mathbf{a}_2 + \dots \quad (20)$$

$$\omega = \omega_0 + \omega_1 + \omega_2 + \dots \quad (21)$$

$\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$ are column vectors and $\omega_0, \omega_1, \omega_2$ are complex eigenfrequencies. Matrices $\mathbf{M}_0, \mathbf{M}_1$ and $\mathbf{M}_2$ are given by:

$$\mathbf{M}_0^{m,p} = \delta_{m,p}\left(1 + \frac{1}{4}\chi_m^{-2}\hat{\Pi}_m(\omega)\right) \quad (22)$$

$$\mathbf{M}_1^{m,p} = \frac{1}{2}\chi_m^{-2}\left[\hat{\Pi}_m(\omega)\alpha_p^m(E) + \hat{\Pi}_p(\omega)\alpha_m^p(E)\right] \quad (23)$$

$$\mathbf{M}_2^{m,p} = \chi_m^{-2}\sum_{n=1}^{\infty}\hat{\Pi}_n(\omega)\alpha_m^n(E)\alpha_p^n(E) \quad (24)$$

Using the relations (19) through (24), we rewrite the dispersion relation (14), collect the terms of orders $\varepsilon^0, \varepsilon^1$ and ε^2 and set the dispersion relation at each order to zero separately. We are specifically interested in the $\mu = 1$ squeezed mode, i.e. the mode which is closest to the $m = 1$ unsqueezed mode with spatial dependence $\cos[k_1(z+L/2)]$. This is the mode which was used and studied in the experiments. The zero'th order dispersion relation is given by:

$$\mathbf{M}_0(\omega_0).\mathbf{a}_0 = 0 \Rightarrow \left(1 + \frac{1}{4}\chi_m^{-2}\hat{\Pi}_m\right)\delta\bar{\phi}_m = 0 \quad (25)$$

Equation (25) is the dispersion relation of an unsqueezed plasma. Since $\mathbf{M}_0$ is a diagonal matrix, each Fourier cosine function in (9) is an eigenmode for an unsqueezed plasma. The zero'th order eigenfrequency ω_0 and the eigenvector are obtained by solving (25). For mode $\mu = 1$ the zero'th order(unsqueezed) eigenvector and the damping rate are:

$$\mathbf{a}_0^T = (1, 0, 0, \dots) \quad (26)$$

$$\omega_0^i = -\frac{\mathrm{Im}\hat{\Pi}_1(\omega_0)}{\partial_\omega \mathrm{Re}\hat{\Pi}_1(\omega_0)} \quad (27)$$

When $\omega_0/k_1 v_T \gg 1$, the zero'th order damping rate ω_0^i, which is the Landau damping of the unsqueezed mode, is exponentially small and $\omega_0 = \omega_0^r$. We obtain ω_1^i and $\mathbf{a}_1$ from the first order dispersion relation:

$$\omega_1^i = -\frac{4\mathrm{Im}\hat{\Pi}_1(\omega_0)\alpha_1^1(E)}{\partial_\omega \mathrm{Re}\hat{\Pi}_1(\omega_0)} \quad (28)$$

$$\mathbf{a}_1^T = (0, \delta\bar{\phi}_2, \delta\bar{\phi}_3, \dots); \quad \delta\bar{\phi}_j = -\frac{\mathbf{M}_1^{j,1}(\omega_0)}{\mathbf{M}_0^{j,j}(\omega_0)}, \quad j > 1 \quad (29)$$

The first order damping rate ω_1^i also turns out to be exponentially small, in the regime where $\omega_0 \gg k_1 v_T$. Thus, both the zero'th order and the first order correction to damping rate are nearly zero and the dominant behavior of the damping rate is of second order with respect to smallness parameter ε. Since $\varepsilon = q\varphi_s/T$ and for small ε we have $\varphi_s \propto |V_{sq}|$, if $\omega_0 \gg k_1 v_T$ damping rate will be proportional to $|V_{sq}|^2$. This behavior is in qualitative agreement with the experimental observations.

We obtain the second order damping rate from the second order dispersion relation. For $\omega_0 \gg k_1 v_T$ we have:

$$\omega_2^i = -\frac{4\sum_{n=1}^{\infty} \mathrm{Im}\hat{\Pi}_n(\omega_0)(\alpha_1^n(E))^2}{\partial_\omega \mathrm{Re}\hat{\Pi}_1(\omega_0)} + \frac{4\chi_1^4}{\partial_\omega \mathrm{Re}\hat{\Pi}_1(\omega_0)} \sum_{j=2}^{\infty} \frac{1}{\chi_j^2} \mathrm{Im}\left[\frac{(\mathbf{M}_1^{1,j}(\omega_0))^2}{\mathbf{M}_0^{j,j}(\omega_0)}\right]$$

The first term on the RHS, which we call γ_1, is the contribution to damping rate due to $m = 1$ unsqueezed mode(cosine in position space). Particles slow down as they pass the squeeze and thus, no longer see this mode as a simple cosine along their bounce orbits, as a function of their angle variables. As a result, the $m = 1$ unsqueezed mode has nonzero Fourier terms in angle variable space. Particles with bounce frequency $\omega_b = \omega/n$ will resonantly interact with the n'th Fourier term and thus, enhance the damping rate of the mode.

Moreover the squeezed eigenmode is a superposition of unsqueezed eigenmodes, since from (29), elements of $\mathbf{a}_1$ are nonzero. Therefore, the shape of the mode potential is no longer a simple cosine in position space and consists of higher harmonics in z, which are all oscillating at the same frequency ω. The contribution to the damping rate given by second term on the RHS of (30), which we call γ_2, is due to the damping of these higher harmonics which are coupled to the $m = 1$ unsqueezed mode. In figure (2) we compare the damping rate calculated from our compter simulation results to the analytically calculated damping rates. We chose our parameters in the regimes where phase velociy was much greater than thermal velocity, so that unsqueezed Landau damping is exponentially small. For our analytical results, we depicted the contribution to second order(in ε) damping rate from γ_1(circles) and γ_2(squares) separately, as well as the total second order damping rate(diamonds). Computer simulation results are depicted with triangles. We can see that damping rate resulting from γ_2 is, in most parts, at least an order of magnitude greater than damping rate from γ_1. As the amplitude of squeeze potential is increased, analytically calculated damping rates become smaller than the computer simulation results and deviate from square dependence behavior on $|V_{sq}|^2$. This is caused by the fact that $q\varphi_s/T$ is no longer a small value, and thus our perturbation method is no longer valid. Also the population of trapped particles, which was not accounted for, becomes larger as φ_s grows and resonant trapped paticle-wave interaction becomes significant, further enhancing the mode damping rate.

CONCLUSION

The presence of a squeeze potential results in additional resonant wave-particle interactions at bounce frequencies $\omega_b = \omega/n$, which enhances the mode damping rate of Trivelpiece-Gould modes. There are two different reasons for these extra resonances to be generated: i)The squeeze potential modifies the unperturbed orbits of particles in such a way that a single cosine wave in position space is seen by the particles (as a func-

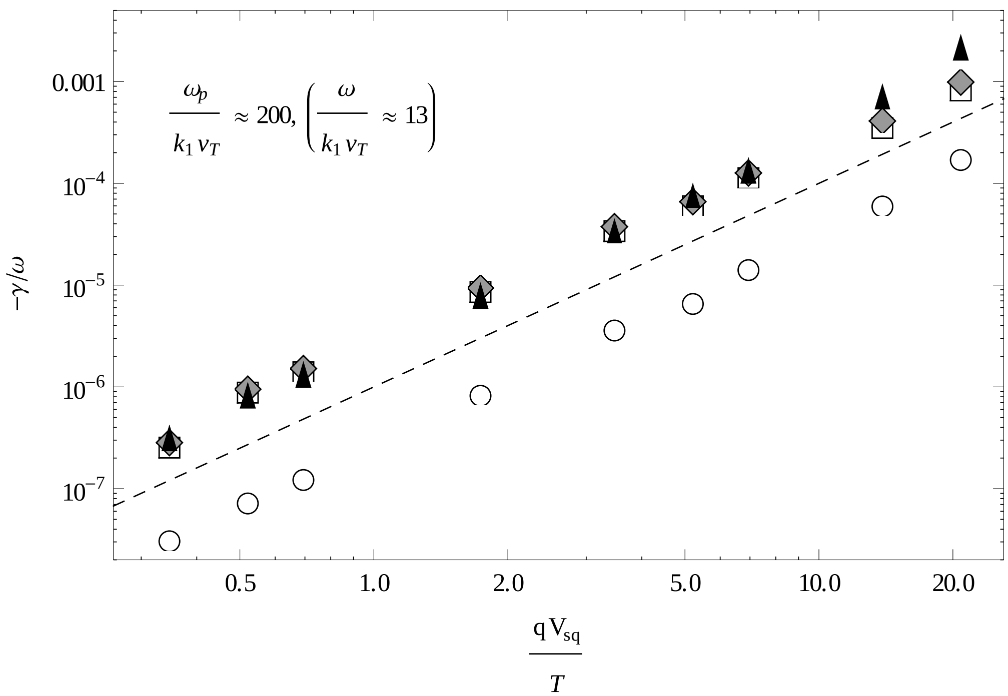

FIGURE 2. Mode damping rate vs. squeeze potential. Analytically calculated damping rate is shown in terms of the value from γ_1 with circles, γ_2 with squares, and their total sum $\gamma_1 + \gamma_2$ with diamonds. Computer simulation results is shown with triangles. Dashed black line is $f(x) = 10^-6 * x^2$, depicted for comparison.

tion of time) as perturbed, with higher harmonics with amplitude of order $\varepsilon = q\varphi_s/T$ added to the wave. ii) The shape of mode potential in position space is also modified and contains higher harmonics of amplitude ε. Our analysis shows that in the regime where $q\varphi_s/T \ll 1$, and $\omega/k_1 v_T \gg 1$, the mode damping rate has a square dependence on the amplitude of the applied squeeze potential $|V_{sq}|$. This behavior is consistent with the experimental results. We compared our analytical results to computer simulations, details of which will be discussed in a future publication.

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