Non-Linear Plasma Wave Decay to Longer Wavelength

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Abstract. We measure the decay of plasma waves to longer wavelengths, for both "standard" Trivelpiece-Gould waves with $v_\phi \gg \bar{v}$, and for the lower phase velocity "EAW" modes with $v_\phi \sim \bar{v}$. These are θ-symmetric standing modes on pure ion or pure electron plasma columns with discrete wavenumbers $k_z = m_z \pi/L_p$. A large amplitude $m_z = 2$ Trivelpiece-Gould wave causes phase-locked exponential growth of the $m_z = 1$ wave when they are near resonant, at growth rates $\Gamma_e \propto \delta n_2/n$ consistent with cold fluid theory. For larger detuning $\Delta \equiv 2f_1 - f_2$, mode amplitude A_1 is observed to "bounce" at rate Δf, with amplitude excursions $\Delta A_1 \propto \delta n_2/n$ also consistent with cold fluid theory; but A_1 often exhibits a slower overall *growth*, as yet unexplained by theory. In contrast, a large amplitude $m_z = 2$ EAW mode generally causes either strong phase-locked $m_z = 1$ growth or no growth at all, apparently because the EAW "frequency fungibility" enables $\Delta f = 0$, and EAW mode damping is strong until the velocity distribution $F(v_{phase})$ is "flattened."

INTRODUCTION

Plasma wave decay instability from short to long wavelengths has been previously observed in pure electron plasmas [1,2]. Here, we quantify the non-linear coupling rates of standing plasma wave $k_z = m_z \pi/L_p$ from short wavelength $m_z = 2$ mode to a longer wavelength $m_z = 1$ mode for both Trivelpiece-Gould (TG) and Electron Acoustic Waves (EAW) on pure ion and electron plasmas. The dispersion relations of these waves are nearly acoustic, with an off acoustic frequency deviation described by the detuning $\Delta \equiv 2f_1 - f_2$. We find this frequency detuning to be fundamental to the overall behavior of the non-linear wave coupling. The measured decay rates are compared with prediction of cold fluid theory and Vlasov theory. At large $m_z = 2$ amplitudes TG waves and all EAWs exhibit exponential decay from short to long wavelengths. In contrast, for TG waves with a smaller $m_z = 2$ amplitudes the non-linear coupling creates amplitude oscillation of the $m_z = 1$ mode near the detuning frequency Δ.

These two regimes and the strength of the non-linear coupling rates are consistent with predictions of cold fluid and Vlasov theory, and numerical Vlasov simulations.

Higaki [1] observed that the decay threshold depends on the plasma temperature, decreasing as the temperature increases from 0.2 eV < T < 0.8 eV. However, contrasting with Higaki results, the measured strength of the coupling rates seem to be independent of temperature (see figure 6) suggesting that trapped particles do not play a significant role in the decay process.

TRIVELPIECE-GOULD AND "ELECTRON ACOUSTIC" PLASMA WAVES

Trivelpiece-Gould waves are the "regular, upper-branch" plasma waves with shielding from the cylindrical wall causing frequencies below the plasma frequency $\omega_p/2\pi$. According to cold fluid theory, azimuthally symmetric standing plasma waves in the trapped cylindrical plasmas have dispersion relation:

$$\omega_{TG} = \omega_p \frac{k_z}{\sqrt{k_z^2 + k_\perp^2}}$$

(eq. 1)

where $k_z = m_z \pi / L_p$, with $m_z = 1, 2, 3, ...$ representing the number of "half-wavelength" in the plasma. The perpendicular wave number can be approximated by $k_\perp \approx R_p^{-1}[2/\ln(R_w/R_p)]^{1/2}$. The exact $k_\perp$ is given by a complicated ratio of Bessel functions, resulting from the requirement that the potential vanish at the conducting wall [3]. For a magnesium ion plasma, the plasma frequency is $\omega_p/2\pi \cong 140. kHz \cdot n_7^{1/2}$, where n_7 is the density in unit of 10^7 cm^{-3}. It is worth noting that for long and thin plasmas ($R_p \ll R_w$) the dispersion relation is almost acoustic since $k_z \ll k_\perp$.

Electron Acoustic Waves (EAW) [2,4] are the lower branch of plasma waves with a slow phase velocity. Typically $\omega_{EAW}/k_z \approx 1.4 \, \overline{v}$; in contrast TG waves have $\omega_{TG}/k_z \geq 3\overline{v}$. The EAW name comes from the neutral plasma community, where the lower branch of electron plasma waves is acoustic even without boundary walls and the mobile electrons give an acoustic response. Analogous EAW modes are observed in pure electron and separately pure ion plasmas (where the name is somewhat misleading since electron are not involved). The EAW is non-linear so as to flatten the particle distribution to avoid strong Landau damping, but it can exist at small amplitude. Figure 1 shows the typical phase velocity of an EAW and TG wave on the particle velocity distribution.
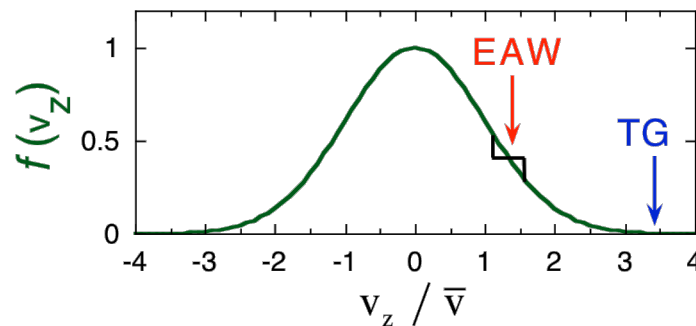
FIGURE 1. *Particle distribution and phase velocity of plasma waves.*

The two dispersion branches for a typical "warm" magnesium ion plasma of density $n = 0.67 \cdot 10^7 cm^{-3}$ ($n_7 = 0.67$) temperature T = 0.5eV, radius $R_p = 0.73$, length L_p= 8.9cm, Debye length λ_D =0.2cm is shown on figure 2. The dispersion relation of similar pure ion plasmas has been previously measured [4] and found to be correctly describe by the Vlasov Poisson theory used calculated the curve of figure 2. The upper green circle represents the $m_z = 2$ TG wave which decays to the lower green circle $m_z = 1$, similarly the red circles show the decay of the EAW wave to longer wavelength.
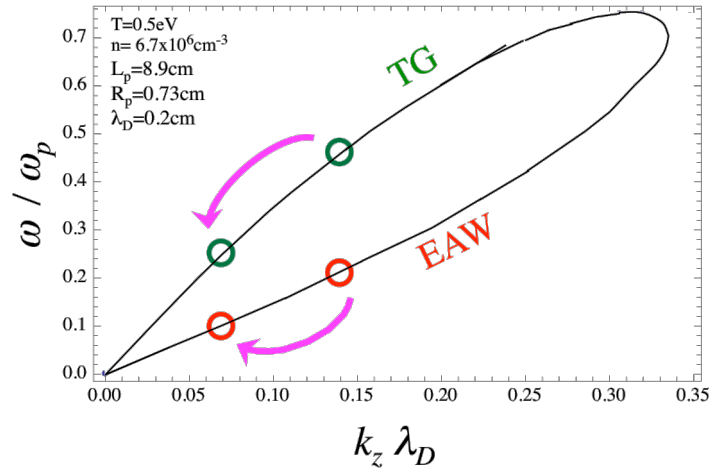
FIGURE 2. *Dispersion relation of plasma waves in trapped plasma.*

TRIVELPIECE GOULD WAVE NON-LINEAR DECAY

Most of the non-linear decay results presented here are obtained with a magnesium ion plasma contained in a Penning Malmberg trap [5] at a magnetic field of 3 Tesla, and a wall radius R_w = 2.86cm. The ion density is maintained at typically $n_7 = 2$ with a weak rotating wall perturbation [6]. For cold plasma, the plasma radius is typically $R_p \cong 0.4$ cm and the plasma length is varied from 10 to 12 cm. Laser cooling and cyclotron heating control the plasma temperature in the range of 0.001 eV < T < 1 eV.

The TG waves are excited by a sinusoidal burst of 10 to 40 cycles at the m_z = 2 frequency. To insure consistent initial conditions a 5% m_z = 1 seed is often mixed with the driving burst. A separate electrode detects the waves, and the wall signal is recorded and Fourier analyzed to identify the various frequency components. The detected wall signal is then fit to a sum of sinusoidal components: the large m_z = 2 wave at frequency f_2 has harmonics at frequencies $n_i f_2$ and the small m_z = 1 wave at frequency f_1 and its eventual harmonics at frequencies $n_i f_1$ as wave 1 is growing. The fits are performed in time slices of typically 10 cycles of wave 2 for slow growth or oscillatory behavior. For rapid exponential growth of wave 1, the fitting function of wave 1 is an exponentially growing sine wave.

Figure 3 show the evolution of wave 2 amplitude $A_2 \equiv \delta n_2/n$ and the initially small wave 1 amplitude $A_1 \equiv \delta n_1/n$. At early time, the long wavelength (m_z = 1) wave grows exponentially while the amplitude of the large m_z = 2 wave remain almost constant $A_2 \sim 0.6 \to 0.5$. Wave 1 grows with an initial exponential growth rate $\Gamma_e = 14,600./sec$. For this data, the frequencies of the waves are of the order of $f_1 \sim 15 \, kHz$, and $f_2 \sim 30 \, kHz$; that

is, wave 1 grows by one decade in ~ 6 cycles of wave 2. The exact frequencies of wave 1 and 2 change with time due to non-linear frequency shifts. The two waves are essentially "phased-locked" during the decay, and the phase difference $\Delta\phi = 2\phi_1 - \phi_2$ change by ~π when the two waves are of comparable amplitude and the direction of energy flow reverses.

Figure 4 is similar to figure 3 but the initial amplitude of wave 2 is smaller: $A_2 = 0.25$. Here wave 1 does not grow exponentially, but oscillates at the detuning frequency $2f_1 - f_2$. That is, wave 1 is driven up while in-phase with wave 2 and driven down while out-of-phase with wave 2.
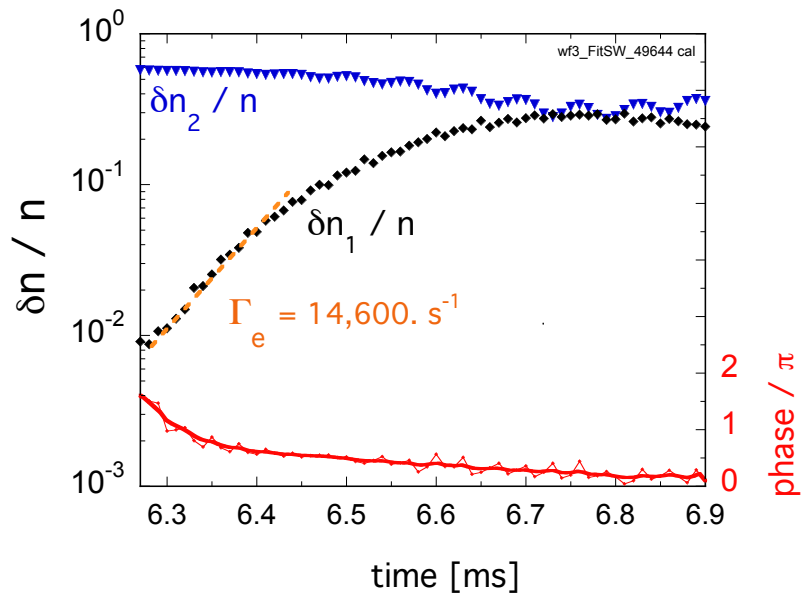

FIGURE 3. *Amplitude of TG wave 2 and wave 1. Also plotted is the relative phase for initial $A_2 = 0.6$.*
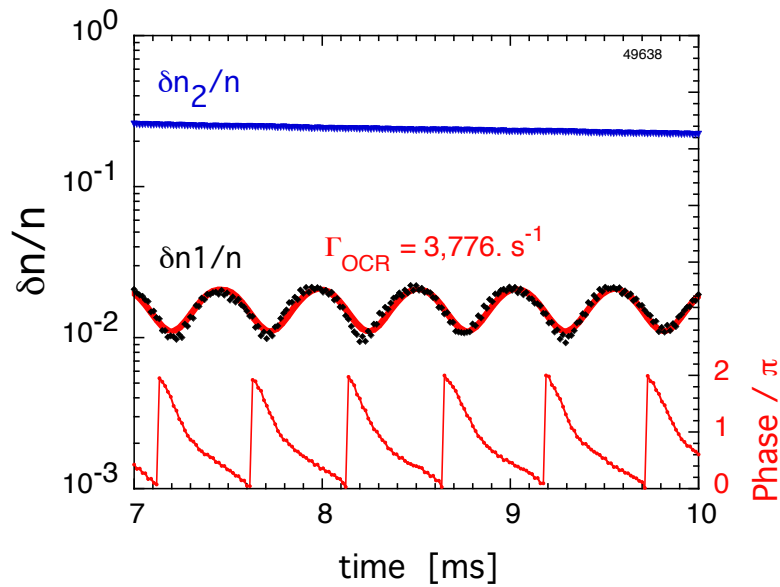

FIGURE 4. *Same as figure 3 but smaller initial amplitude A2 =0.25.*

The oscillations of A_1 can be described as $A_1(t) = \langle A \rangle + A_b \sin(\omega_b t + \phi)$ as shown by the solid red curve on figure 4 fit as $\langle A \rangle = 0.016$, $A_b = 0.005$ and $\omega_b = 2\pi \cdot 1.93\ kHz$. The measured "bounce" frequency $\omega_b / 2\pi$ is the same as the instantaneous detuning $2f_1 - f_2$. The "oscillatory coupling rate" is defined to be:

$$\Gamma_{OCR} \equiv \frac{\dot{A}_1}{A_1} = \frac{A_b\,\omega_b}{\langle A \rangle}$$

(eq. 2)

Γ_{OCR} is the rate at which energy is exchange between wave 2 and wave 1.

Simple Theory Model

Extensive cold fluid and Valsov theory analysis has been developed to describe non-linear wave interactions in various regimes [7]. Here we present a simplified version of the cold fluid theory considering two waves. In the frame of wave 2, the density fluctuation N_1 associated with wave 1, is described by:

$$\ddot{N}_1 = \left[\Gamma_0^2 - \left(\Delta/2\right)^2\right] N_1$$

(eq. 3)

Where Γ_0 is the non-linear coupling rate in the absence of detuning, given by

$$\Gamma_0 = \frac{\delta n_2}{n}\,\frac{R}{4}\,\frac{\omega_1 + \omega_2}{2}$$

(eq. 4)

with $R = 0.85$ accounting for the non-radial uniformity of the wave electric field, the factor of 4 accounts for two travelling waves (i.e. standing wave), and Δ is the detuning at small amplitude as previously defined. Equation 3 has an exponentially growing solution when $\Gamma_0 > \Delta/2$, and an oscillating solution when $\Gamma_0 < \Delta/2$. In general, the detuning Δ creates a threshold for exponential growth.

Experiment

Figure 5 shows the measured non-linear coupling rate Γ_{OCR} and the growth rate Γ_e both are normalized by Δ in order to plot the result from different plasma geometries. The horizontal axis represents the normalized amplitude of wave 2. The open symbols are measurements of the oscillatory coupling rate, and the solid symbols are from measurement of the exponential growth of wave 1. Also shown are data from an electron plasma, which is long and thin (L_p=48cm, R_p=1.2cm), and consequently has a small detuning $\Delta/f_1 = 2\%$. All the electron data are in the exponential growth regime. The numerical Vlasov simulations of the plasma wave decay are shown with square where the lower right corner filled-in indicates oscillatory coupling and top right corner filled-in indicates exponential growth. The dashed line is Γ_0, the coupling strength in the absence of detuning, and the solid line is the result of fluid theory considering two standing plasma waves (Eq. 3). When the normalized amplitude of wave 2 is

equal to 1, the fluid theory predicts a dip in the growth rate corresponding to $\Gamma_0 = \Delta/2$. At present it is an open question why the experimental measurement do not exhibit a dip at a normalized amplitude of 1 predicted by cold fluid theory.

Overall the Trivelpiece-Gould wave non-linear coupling rates are consistent with cold fluid theory in both the oscillatory and exponential growth regimes.
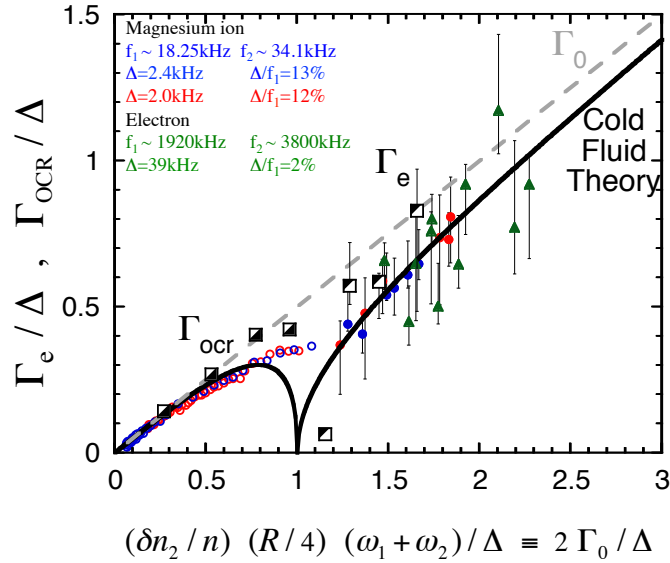

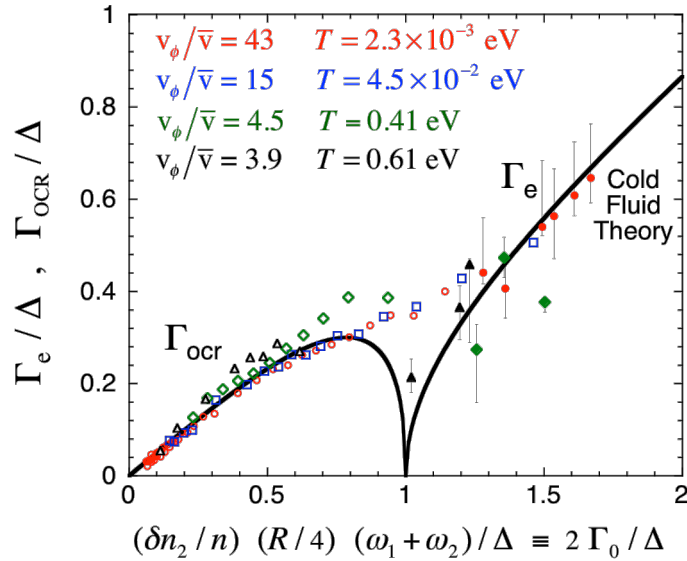**FIGURE 5.** *Theory predicted and measured non-linear coupling rate.*

FIGURE 6. *Non-linear coupling rate for various* $v_\phi/\overline{v}$.

Changing the number of particles present at the phase velocity by changing the plasma temperature appears to have little effect on the overall non-linear coupling rate. Figure 6 shows the normalized non-linear TG coupling rate for 4 different temperatures resulting in $3.9 < v_\phi/\overline{v} < 43$. Suggesting that the overall non-linear coupling rate is independent of the presence of trapped particles at the phase velocity.

Slow Average Growth

We observe 3 types of non-linear coupling, so far we have described exponential growth for large amplitude of wave 2, and oscillatory coupling for smaller amplitude of wave 2. For intermediate amplitude of wave 2 a slow average growth is observed on top of the oscillatory coupling. Figure 7 shows the amplitude of wave 1 for 3 different amplitudes of wave 2, $\delta n_2/n \cong 0.4$ (exponential growth), $\delta n_2/n \cong 0.3$ (slow average growth), $\delta n_2/n \cong 0.2$ (oscillatory coupling). The slow average growth is temperature dependent and is larger at higher temperature, it is not explained by current theory.
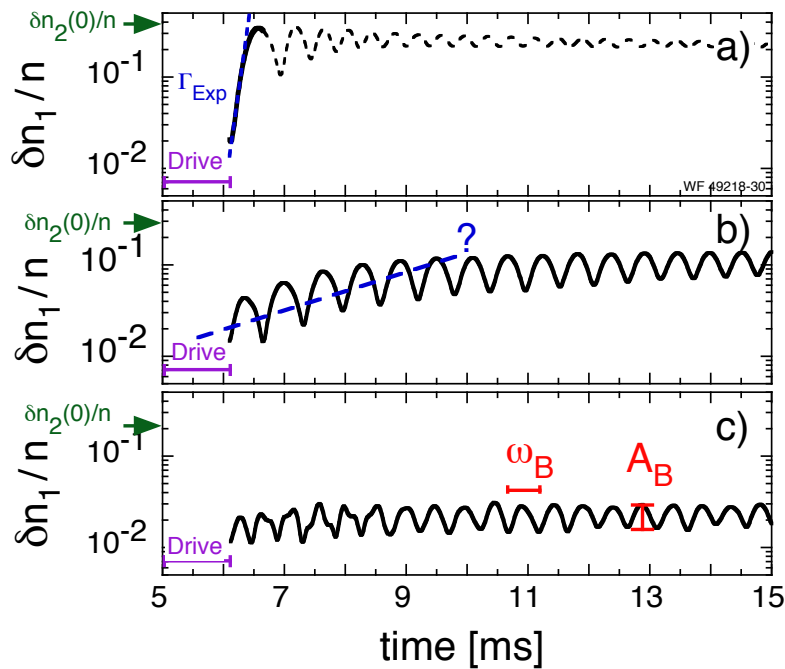

FIGURE 7. *Three types of non-linear coupling: a) $\delta n_2/n \cong 0.4$, b) $\delta n_2/n \cong 0.3$, c) $\delta n_2/n \cong 0.2$.*

ELECTRON ACOUSTIC WAVE NON-LINEAR DECAY

EAWs are harder to excite and typically require a few hundred cycles drive carefully shaped to avoid accidentally driving TG waves. Figure 8 shows exponential growth of wave 1 for an initial amplitude of wave 2 $\delta n_2/n \cong 0.01$ .The measured initial growth rate $\Gamma_e = 735.\,s^{-1}$ and gets smaller as $\delta n_2/n$ decreases. Here, the frequencies of wave 1 and 2 are of the order of $f_1 \sim 8\ kHz,\ f_2 \sim 16\ kHz$. It is worth noting that the relative phase $\Delta\phi = 2\phi_1 - \phi_2$ is constant during the growth of wave 1, since EAW frequencies are "fungible", wave 1 can easily "lock" to half the frequency of wave 2 that is $f_1 = f_2/2$.
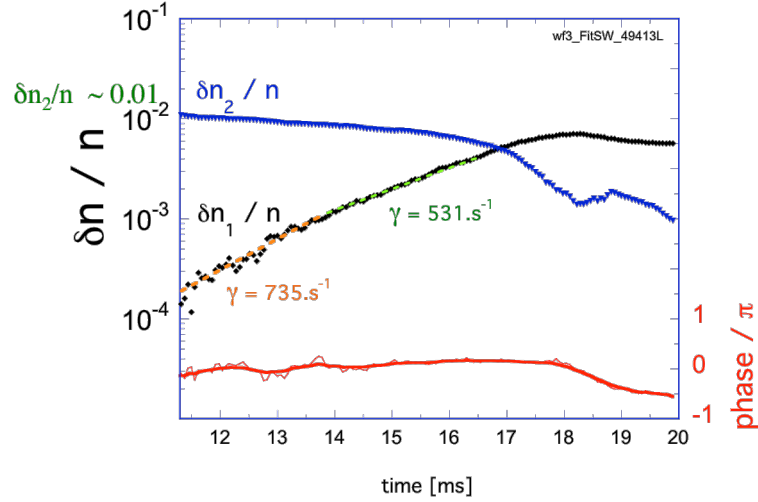
FIGURE 8. *Amplitude of EAW wave 2 and wave 1, also plotted is the relative phase.*

Figure 9 shows the growth rate of EAWs and TG waves on the same plot. We find that the m_z=2 EAW is stable for $\delta n_2/n\ < 0.005$, but that at larger amplitudes the m_z=2 wave decays with exponential growth of the m_z=1 wave. We do not observe oscillatory coupling for small amplitude EAWs. The measured EAW growth rate is correctly described by a Vlasov theory considering two waves [7] shown by the solid line. Also, Vlasov numerical simulations (red square) are in good agreement with the measurements [8]. The EAW growth rate is 4 times larger than the TG coupling rate, probably due to the intrinsic difference between the TG and EAW $\delta n_2/n$. An EAW can have a small $\delta n_2/n$ while still being non-linear since "fast" and "slow" particles travel in opposite direction and almost cancel each other in the total fluid response measured by the electrodes [4].
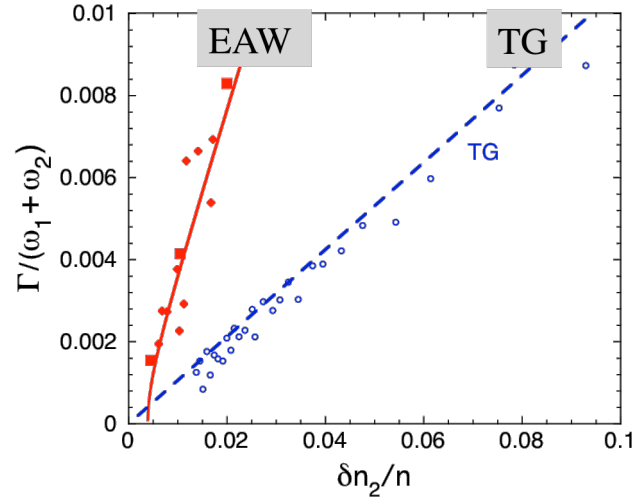
FIGURE 9. *Growth rate of EAW and TG wave plotted versus $\delta n_2/n$.*

SUMMARY

Electron acoustic waves (EAW) and large amplitude TG waves ($\Gamma_0 > \Delta/2$) exhibits exponential growth of the m_z=1 wave. Smaller amplitude TG waves ($\Gamma_0 < \Delta/2$) drives oscillations of the m_z=1 wave amplitude at frequency $2f_1 - f_2$. Cold fluid theory predicts the measured non-linear coupling rate for TG waves. It appears that the overall non-linear coupling rate is independent of the presence of trapped particles at the phase velocity.

Electron acoustic wave decay rate is correctly described by Vlasov theory.

ACKNOWLEDGMENTS

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